Institutional Effectiveness Report

Name of Program/Department:	Department of Mathematics
Year:	2015-2016
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Program Mission Statement

A primary mission of the Department of Mathematics at Francis Marion University is to offer all University students a varied and well-balanced curriculum of undergraduate education in mathematics. In the liberal-arts tradition, the courses in the curriculum teach students to think logically, to analyze problems and solve them appropriately, and to communicate their ideas clearly.

Program Learning Outcomes (PLOs)

Student will be proficient in the elementary computational techniques in the calculus course sequence

Student will be proficient in the ability to understand and construct elementary proofs

Students will be proficient in the use of appropriate technology to solve mathematical problems

Students will be proficient in effectively communicating mathematics in written form and oral presentations

Student Learning Outcomes (SLOs)

SLO 1: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary Computational techniques in the calculus course sequence. (Benchmark = 80%).

SLO 2: Students in Math 230 and 311demonstrate the ability to understand and construct elementary proofs. (Benchmark = 70%).

SLO 3: Students in Math/CIS 212 will demonstrate the ability to use appropriate technology to solve mathematical problems. (Benchmark = 70%).

Assessment Methods

SLO 1: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary Computational techniques in the calculus course sequence. (Benchmark = 80%).

Instructors of Calculus sequence courses (Math 201, 202, 203, 306) and Mathematics Capstone Course (Math 499) provided samples of student solutions to problems or other work that called for students to demonstrate proficiency of basic computational techniques in the calculus sequence. Student solutions were evaluated based on a calculus performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The benchmark was set at 80% for meeting or exceeding faculty expectations.

SLO 2: Students in Math 230 and 311demonstrate the ability to understand and construct elementary proofs. (Benchmark = 70%).

Instructors of Discrete Mathematics (Math 230) and Transitions to Higher Mathematics (Math 311) provided samples of student solutions or relevant problems of other work to demonstrate the ability to understand and construct elementary proofs. Student solutions were evaluated based on a proof performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The benchmark was that 70% of students would meet or exceed faculty expectations.

SLO 3: Students in Math/CIS 212 will demonstrate the ability to use appropriate technology to solve mathematical problems. (Benchmark = 70%).

Instructors of Introduction to Fortran (Math/CS 212) provided samples of student solutions to relevant problems of other work to demonstrate the ability to use appropriate technology to solve mathematical problems. Student solutions were evaluated based on a programming performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The benchmark was that 70% of students would meet or exceed faculty expectations.

Assessment Results

SLO 1: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary Computational techniques in the calculus course sequence. (Benchmark = 80%).

Data collected in Calculus sequence courses indicated an average of 40.87% for students demonstrating proficiency of basic computational techniques. This target was not achieved.

SLO 2: Students in Math 230 and 311demonstrate the ability to understand and construct elementary proofs. (Benchmark = 70%).

Data collected in the Discrete and Transitions to Higher Mathematics courses indicated an average of 69.2% for students demonstrating proficiency in the ability to understand and construct elementary proofs. This target was not achieved.

SLO 3: Students in Math/CIS 212 will demonstrate the ability to use appropriate technology to solve mathematical problems. (Benchmark = 70%).

Data collected in the Introduction to Fortran course indicated that 75% of students demonstrated the ability to use appropriate technology to solve mathematical problems. Since our benchmark was 70%, this target was achieved.

Action Items

SLO 1: Mathematics students at FMU will be proficient in the elementary computational techniques in the calculus course sequence.

Since the target was not achieved, the department made changes to bring about improvement in Student Learning Outcomes. Instructors of calculus courses have allocated more instructional time to elementary computational techniques by including more in-depth content and assessment.

SLO 2: Mathematics students at FMU will demonstrate the ability to understand and construct elementary proofs.

Since the target was not achieved, the department made changes to bring about improvement in Student Learning Outcomes. Instructors of mathematical proofs courses have allocated more instructional time to the understanding and construction of elementary proofs by including more in-depth content and assessment.

SLO 3: Mathematics students at FMU will be able to use appropriate technology to solve mathematical problems. This target was achieved.

Appendices

Additional detail for Student Learning Outcomes

Goal 1: Every student will be proficient in the elementary computational techniques in the calculus course sequence.

Outcome 1: Students will demonstrate competence to calculate derivatives and use them in various applications, such as optimization or related rates problems (Math 201/499).

Outcome 2: Students will demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499).

Outcome 3: Students will demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499).

Outcome 4: Students will demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499).

Outcome 5: Students will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence (Math 499).

Goal 1 Outcome 1

Assessment Problems:

Problems 1 and 2:
Two differentiation problems that require use of at least two of the following differentiation rules: chain rule, product rule and quotient rule. Examples of such problems are as follows:

$$\frac{x\cdot e^x}{\cos x},\quad \frac{x\cdot\cot x}{7^x},\quad \frac{\ln x^2}{x},\quad (2x^3+4x^{-1}+3)^{10}\cdot\log_3 x,\quad x^2\cdot\arctan 5x,\quad \sin(\cos x^4)$$

Calculus Performance rubric for Outcome 1 for Problems 1 and 2:

Score	Criteria
1	• Incorrect approach • Major calculation errors
2	• Properly identified rules required for the solution • Minor calculation errors including algebra and notation
3	• Correct answer with proper notation

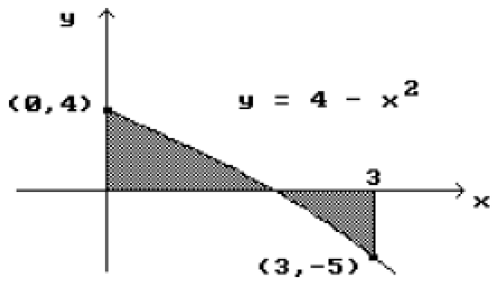
Problem 3:
A farmer wants to put a fence around his crops. His rectangular garden will contain potatoes and carrots. He wants to put a fence around the garden and a piece of fence in the middle, parallel to the short side of the garden, dividing it in half with one section for carrots and the other for potatoes. If he has 300ft of fence, what is the maximum total area his garden can have? (Remember to clearly state knowns and unknowns, draw a picture and write clear notation and equations.)

Calculus Performance rubric for Outcome 1 for Problem 3:

Score	Criteria
1	Any solution in which the all four criteria for a score of 2 were not achieved.
2	• Correct objective and constraint equations • Proper substitution • Proper differentiation and solution for critical points • Partial correct answer (for example, gives correct width but does not solve for area) or correct solution but incorrect final answer due to arithmetic error.
3	• All differentiation/calculations completed correctly • Answer is given with units of measure • Given question was answered • Verification the solution is a maximum

Find the area of the shaded region.

1)



Total sum of 3-5 produces an overall score of 1, 6-7 produces an overall score of 2, and 8-9 produces an overall score of 3.

Goal 1 Outcome 2

Assessment Problems:

Calculus Performance rubric for Outcome 2 for Problem 1:

1 - Does not meet faculty expectations
  - *Integral set up incorrectly.
  - *Integral set up correctly but evaluated incorrectly at a conceptual level such as the incorrect use of the Evaluation Theorem or the absolute value of the integral is not taken.

2 - Meets faculty expectations
  - *Integrals are set up properly.
  - *Integrals are evaluated properly with possible minor calculation errors.
  - *Absolute value is taken to reflect the area of the entire shaded region.

3 - Exceeds faculty expectations
  - *Correct solution with proper work shown and no calculation errors.

Use the substitution formula to evaluate the integral.

2) $\int_{-1}^{0} \frac{2t}{\left(3 + t^2\right)^3}\, dt$

Evaluate the integral.

3) $\int 5x \sin x \, dx$

Calculus Performance rubric for Outcome 2 for Problems 2 and 3:

Score	Criteria
1	• Incorrect approach • Major calculation errors
2	• Properly identified rules required for the solution • Minor calculation errors including algebra and notation
3	• Correct answer with proper notation

Total sum of 3-5 produces an overall score of 1, 6-7 produces an overall score of 2, and 8-9 produces an overall score of 3.

Goal 1 Outcome 3

Assessment Problem:

(a, b) Using either Taylor's Theorem or the Method of Undetermined Coefficients, *derive* a series expansion for the function e^x centered at zero.

(c, d) Using one or more tests for convergence, find the interior of the domain of convergence for the series derived in part (a,b).

(e) Using the series expansion in part (a,b), determine the value of $e^{1.5}$ to at least three digits of accuracy. Tell how many terms you used to obtain the approximation. NOTE: Do not simply report the known value. Report the *approximation.*
Approximation: __________________ *n* (degree on *x* you went to): _____

(f) Compare the result of part (e) with the known value for $e^{1.5}$, which is readily enough obtained on an handheld calculator. Then check the appropriate remark:
____ The approximation and known value are close enough that my work appears to be correct.
____ The approximation and the known value are so far apart that my work appears to be incorrect.

Calculus Performance rubric for Outcome 3:

1 (Not meet expectation) Student does not even know Taylor's Theorem, or the Method of Undetermined Coefficients, or how to use them, and makes no headway in solving the problem.
2 (Meets expectation) Student knows Taylor's Theorem or Method of Undetermined Coefficients and derives a series expansion for *ex*.
3 (Exceeds expectation) Student determines interior of the domain of convergence using the ratio test, and solves the other parts of the problem.

Goal 1 Outcome 4

Assessment Problem:

Let $f \in \mathbb{R}^2 \to \mathbb{R}^1$ with $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y + 2$. Compute the following functions, numbers, vector, and equation by hand.

(a) $\frac{\partial}{\partial x} f(x,y) \quad =$

(b) $eval_{\,(x,y)=(1,2)} \frac{\partial}{\partial x} f(x,y) \quad =$

(c) $\frac{\partial}{\partial y} f(x,y) \quad =$

(d) $eval_{\,(x,y)=(1,2)} \frac{\partial}{\partial y} f(x,y) \quad =$

(e) $\nabla f(1,2) \quad =$

(f, g) Find the equation of the plane Π tangent to the surface $\{z = f(x, y)\}$ through the point (1, 2, 21).

Calculus Performance rubric for Outcome 4:

1 (Not meet expectation) Student cannot find partial derivatives and evaluate them at the point specified.
2 (Meets expectation) Student can find partial derivatives and evaluate them at the point specified.
3 (Exceeds expectation) Student goes on to find the gradient and equation of the tangent plane.

Goal 1 Outcome 5

How would you respond to the following statement: I am proficient in elementary computational techniques in the calculus.

Disagree *Agree* *Strongly Agree*

Goal 2: Every student will develop the ability to understand and construct elementary proofs.

Outcome 1: Students will be able to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311).

Outcome 2: Students will be able to write elementary proofs (Math 230/311).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs (Math 499).

Goal 2 Outcome 1

Assessment Problem:

For each proposition stated, if the proof given is in fact a valid proof, identify the method of proof used (Direct, Indirect Contrapositive, Contradiction, or Induction). If there is an error in the proposed proof, find and describe the problem. An error could be any of the following: the use of a fallacy argument, an algebraic error, the misuse of a definition, an incomplete proof, and so on.

Proposition 1. *If x and y are integers which are either both even or both odd, then $x - y$ is even.*

Proof. Let x and y be integers. We consider two cases, according to whether x and y are both even or both odd.

Case 1: x and y are both even. Let $x = 6$ and $y = 2$, which are are both even. Then $x - y = 4$, which is even.

Case 2: x and y are both odd. Let $x = 7$ and $y = 1$, which are both odd. Then $x - y = 6$, which is even. □

Proposition 2. *If m is an even integer and n is an odd integer, then $3m + 5n$ is odd.*

Proof. Let m be an even integer and let n be an odd integer. Then $m = 2k$ and $n = 2k + 1$, for some $k \in \mathbb{Z}$. Therefore, we have

$$\begin{aligned} 3m + 5n &= 3(2k) + 5(2k + 1) \\ &= 6k + 10k + 5 \\ &= 16k + 5 \\ &= 2(8k + 2) + 1. \end{aligned}$$

Since $8k + 2$ is an integer, it follows that $3m + 5n$ is odd. □

Proposition 3. *For all $x \in \mathbb{R}$, $|x| = 2$ if and only if $x^2 - 4 = 0$.*

Proof. Case 1: Consider any $x \in \mathbb{R}$ for which $|x| = 2$. Then $|x|^2 = 2^2 = 4$. As $|x|^2 = x^2$ for any $x \in \mathbb{R}$, it follows that $x^2 - 4 = 4 - 4 = 0$.

Case 2: Consider any $x \in \mathbb{R}$ for which $x^2 - 4 \neq 0$. Then $x^2 \neq 4$, and so $\sqrt{x^2} \neq \sqrt{4}$. This implies $|x| \neq 2$, and so this case follows by contrapositive. $\square$

Proposition 4. *For all $x \in \mathbb{Z}$, if $x^2 + 2x$ is even then x is even.*

Proof. Consider any $x \in \mathbb{Z}$ such that x is odd. Then $x = 2k + 1$ for some $k \in \mathbb{Z}$. It follows that

$$\begin{aligned} x^2 + 2x = (2k+1)^2 + 2(2k+1) &= 4k^2 + 4k + 1 + 4k + 2 \\ &= (4k^2 + 8k + 2) + 1 \\ &= 2(2k^2 + 4k + 1) + 1. \end{aligned}$$

Therefore, $x^2 + 2x$ is an odd integer, and the proposition follows. $\square$

Proposition 5. *For all $n \in \mathbb{N}$, $\sum_{j=1}^{n}(2j-1) = n^2$.*

Proof. If $n = 1$, then

$$\sum_{j=1}^{n}(2j-1) = 2(1) - 1 = 1 = 1^2 = n^2.$$

Thus, the statement is true when $n = 1$. Now suppose that the statement is true when $n = k$ for any $k \geq 1$, and thus

$$\sum_{j=1}^{k}(2j-1) = k^2.(*)$$

Then for $n = k + 1$, we have

$$\begin{aligned}
\sum_{j=1}^{n}(2j-1) = \sum_{j=1}^{k+1}(2j-1) &= \sum_{j=1}^{k}(2j-1) + \sum_{j=k+1}^{k+1}(2j-1) \\
&= \sum_{j=1}^{k}(2j-1) + [(2(k+1)-1] \\
&= k^2 + [2(k+1)-1] \qquad \text{by } (*) \text{ above} \\
&= k^2 + 2k + 1 = (k+1)^2 = n^2.
\end{aligned}$$

Thus, if the statement is true when $n = k$ for any $k \geq 1$, the statement has also been shown to be true for $n = k+1$. Therefore, the statement must be true for all $n \in \mathbb{N}$. □

Proof Performance rubric for Outcome 1:

Outcome 1
(1) 0-2 correctly identified proposition(s) does not meet faculty expectations.
(2) 3-4 correctly identified propositions meets faculty expectations.
(3) 5 correctly identified propositions exceeds faculty expectations.

Goal 2 Outcome 2

Assessment Problem:

Provide mathematically correct, properly written proofs for the given statements, using the indicated proof technique if so directed.

Assessment Problem: Provide mathematically correct, properly written proofs for the given statements.

(1) Let X be a non-empty set, and let A, B, and C be subsets of X. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(2) For all $x, y \in \mathbb{Z}$, prove that $x + 3y$ is even if and only if x and y are either both even or both odd.

(3) Prove that, for every positive integer n,

$$\sum_{j=1}^{n} 4j + 6 = 2n(n+4).$$

(4) Prove that, for any $x \in \mathbb{R}$, if $x^2(x+1)$ is irrational then x is irrational.

Proof Performance rubric for Outcome 2:

Problems 1, 3, and 4 count for 1 proof and Problem 2 counts for 2 proofs:
(1) 0-2 proven correctly: does not meet faculty expectations.
(2) 3-4 proven correctly: meets faculty expectations.
(3) 5 proven correctly: exceeds faculty expectations.

Goal 2 Outcome 3

How would you respond to the following statement: My ability to understand and construct elementary proofs is satisfactory.

Disagree *Agree* *Strongly Agree*

Goal 3: Students will be able to use appropriate technology to solve mathematical problems.

Outcome 1: Students will be able to read computer programs that model various mathematical applications (Math/CS 212).

Outcome 2: Students will be able to write computer programs that model various mathematical applications (Math/CS 212).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems (Math 499).

Goal 3 Outcome 1

Assessment Problem: Students will be given two assessment questions, one multiple-choice and the other free-response, to assess their abilities to read computer programs.

1. What is the output of the following code?

```
REAL :: x = 2.3

IF (x <= 2.1  .OR.  x >= 2.2) THEN
   WRITE (*, *)  2*x
ELSE IF (x == 2.3) THEN
   WRITE (*, *)  3*x
ELSE
   WRITE (*, *)  4*x
END IF
```

2. Consider the following output:

```
1 1 1 1 1
2 2 2 2
3 3 3
4 4
5
```

Which of the following code will produce this output?

(A)
```
DO j = 1,  5,  1
  DO k = 1,  5,  1
     WRITE (*,  ‘(1X, I1)’,  ADVANCE =‘NO’)   j
  END DO
  WRITE (*, *)
END DO
```

(B)
```
DO j = 1,  5,  1
  DO k = 1,  j,  1
     WRITE (*,  '(1X, I1)',  ADVANCE ='NO')   j
  END DO
  WRITE (*, *)
END DO
```

(C)
```
DO j = 1,  5,  1
  DO k = 5,  1,  -1
     WRITE (*,  '(1X, I1)',  ADVANCE ='NO')   j
  END DO
  WRITE (*, *)
END DO
```

(D)
```
DO j = 1,  5,  1
  DO k = 5,  j,  -1
     WRITE (*,  '(1X, I1)',  ADVANCE ='NO')   j
  END DO
  WRITE (*, *)
END DO
```

(E)
```
DO j = 1,  5,  1
  DO k = j,  5,  1
    WRITE (*,  '(1X, I1)',  ADVANCE ='NO')   j
  END DO
  WRITE (*, *)
END DO
```

Programming Performance rubric for Outcome 1:

1 – Does not meet faculty expectations
- Neither of the two assessment questions is answered correctly.

2 – Meets faculty expectations
- Exactly one of the two assessment questions is answered correctly.

3 – Exceeds faculty expectations
- Both of the two assessment questions are answered correctly.

Goal 3 Outcome 2

Assessment Problem: Students will be asked to write a computer problem to solve a mathematical problem.

Write a DO loop that finds and displays the maximum and minimum values of $f(x) = 7x - 3x$^2 when $f(x)$ is evaluated at $x = 0.1, 0.2, 0.3, \dots, 5.0$. Also display the x values where the maximum and minimum occur. (For instance, the output could be: f has the maximum 100 at $x = 2$ and the minimum -100 at $x = 3$.)

Programming Performance rubric for Outcome 2:

1 – Does not meet faculty expectations

- Response contains major logical and/or syntax errors.

2 – Meets faculty expectations

- Response contains minor logical and/or syntax errors.

3 – Exceeds faculty expectations

- Response contains no logical errors and at most two minor syntax errors.

Goal 3 Outcome 3

How would you respond to the following statement: My ability to use appropriate technology to solve mathematical problems is satisfactory.

Disagree *Agree* *Strongly Agree*

Goal 4: Students will develop and be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Students will communicate mathematics in a written presentation (Math 499).

Outcome 2: Students will communicate mathematics in an oral presentation (Math 499).

Outcome 3: Secondary education students will demonstrate applications of various strategies and tools in the teaching mathematical concepts (Student Teaching).

Outcome 4: Students will respond to a statement concerning their confidence in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499).

Goal 4 Outcome 1

Assessment Problem:

Students will submit a report and/or display a poster presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

1. The student clearly states the mathematical topic to be examined.
2. The report/poster shows an understanding of the proof or methods used.
3. The student is able to extend results, hypothesize ramifications of the result, and/or apply the result in the appropriate setting.

Communication Performance rubric for Outcome 1:

Failure in any of the above requirement will result in a 1. Satisfactory completion of all three requirements will result in a 2.

Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional poster presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, and acceptable display formatting.

Goal 4 Outcome 2

Assessment Problem:

Students will make a presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

1. The student has a clear understanding of what is set to be proved.
2. The student shows an understanding of the proof by successfully summarizing methods.
3. The student is able hypothesize ramifications of the theorem and/or apply the theorem in the appropriate setting.

Communication Performance rubric for Outcome 2:

Failure in any of the above requirements will result in a 1. Satisfactory completion of all three requirements will result in a 2.
Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, acceptable degrees of audience interaction, and possibly extend results.

Goal 4 Outcome 3

Assessment Problem:

Students will write a lesson plan describing in detail the objectives, methods, and outcomes of a class lesson.

Communication Performance rubric for Outcome 3:

1 – Does not meet faculty expectations

*Lesson plan did not contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles.

* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles in a limited fashion. Accommodations contained less than sufficient details/steps.

2 – Meets faculty expectations

*Lesson plan contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles. Accommodations contained sufficient details/steps.

3 –Exceeds faculty expectations

* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles. Accommodations contained more than sufficient details/steps.

Goal 4 Outcome 4

How would you respond to the following statement: I am able to effectively communicate mathematics in written form and oral presentations.

Disagree *Agree* *Strongly Agree*