THE · 41ST · PEE – DEE · REGIONAL · HIGH – SCHOOL
MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED · BY · FRANCIS · MARION · UNIVERSITY
MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER
TUESDAY · 2017 · DECEMBER · 05

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name in the space below
as you wish it to appear if you are given an award.

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 350 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	**Acceptable**
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$

If a problem specifies that an answer is to be rounded in a certain way, then you *must* round in order to receive credit.

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↑ Name. (Print neatly and fully.)	Page **1**. (# 1,2,3,4,5)	Page **2**. (# 6, 7, 8, 9, 10)
	Page **3**. (# 11, 12, 13, 14)	
↑ High School. * Used only in tie-breaking.	Page **5**. (#15, 16, 17, 18)	
Awards	**Total Correct**	**Weighted Sum***

1. Evaluate 30 + 25. *Don't let the easy ones stump you.*

2. Evaluate 25 – 48.

3. What is the remainder when 48 is divided by 7? *Answer as a positive integer.*

4. Mary received a package 48 days before Christmas. Since November has 30 days, on which date of November did Mary receive her package? (*Note: One* day before Christmas is December 24.)

5. Christmas (= December 25) falls on a Monday this year. On which day of the week did Mary receive her package?

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to **Problem 1:**	Answer to **Problem 2:**	Answer to **Problem 3:**	Answer to **Problem 4:**	Answer to **Problem 5:**
			November	

The set of positive integers is the set $\{1, 2, 3, \cdots\}$. To the right is a table of squares, cubes, and sixth powers of those positive integers between 1 and 10. It may be of use in solving some or all of the problems on this page.

x	x^2	x^3	x^6
1	1	1	1
2	4	8	64
3	9	27	729
4	16	64	4,096
5	25	125	15,625
6	36	216	46,656
7	49	343	117,649
8	64	512	262,144
9	81	729	531,441
10	100	1,000	1,000,000

6. Evaluate $\sqrt{117{,}649}$.

7. Only one positive real number satisfies the following equation. Find it:

$$\sqrt{x} = 9 \cdot \sqrt[3]{x}$$

8. Two positive real numbers satisfy the following equation. Find them *both*:

$$\sqrt{x} + 15 \cdot \sqrt[6]{x} = 8 \cdot \sqrt[3]{x}$$

9. Yet a third real number (not necessarily positive) satisfies the equation in Problem 8. Find it.

10. The number $(2/3)^6$, for example, has been calculated to equal 0.087791495⋯, and when rounded expertly to three significant digits is 0.0878. What is the number $\left(\frac{3}{5}\right)^6$, rounded expertly to three significant digits? *You must answer to the accuracy specified, neither more accurately nor less accurately.*

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 6:	**Answer to Problem 7:**	**Answer to Problem 8:**	**Answer to Problem 9:**	**Answer to Problem 10:**
	$x =$	$x =$ or $x =$	$x =$	

A table of squares is provided on this page. It may be of assistance, depending on how you go about solving these problems. Below are provided three copies of a diagram for your use. The boxes are schematic; their sides are not proportioned to the solution. The next page is blank as a place for you to put your work.

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

11. Mary's package was in the shape of a rectangular parallelapiped. (That is just a plain rectangular box, in normal language.) The box was one inch longer than it was wide, and its height was ½ inch less than its width. Its surface area was 59 square inches. What was the volume of the box?

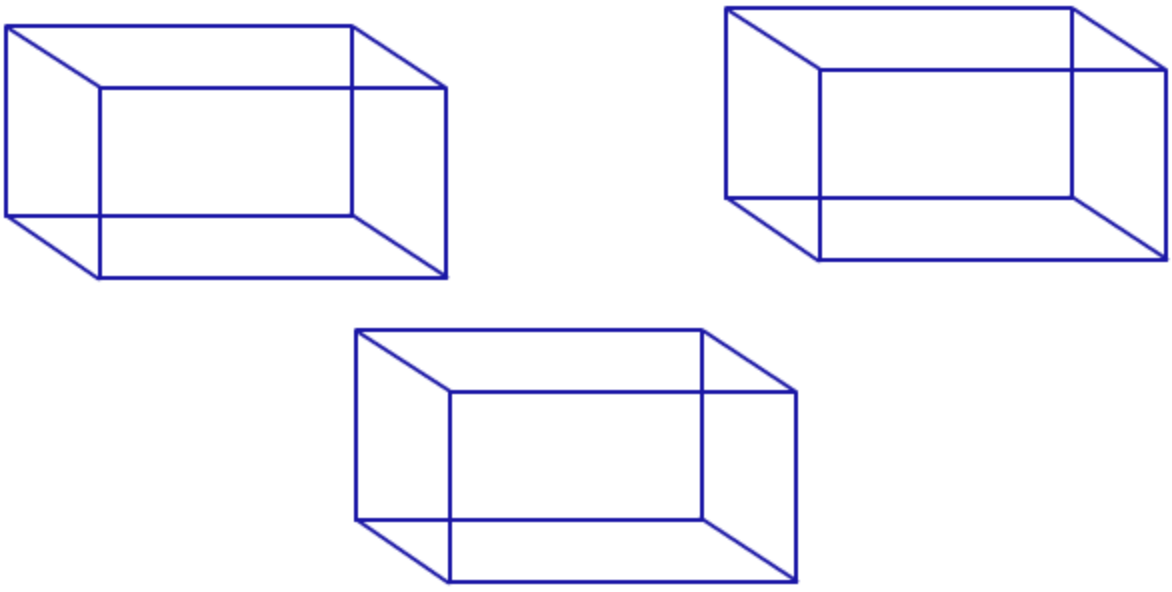
12. What was the length of the diagonal from one vertex of Mary's box to the vertex furthest away from it? (The word "vertex" is just a fancy name for "corner.") *Be careful to simplify your answer per the instructions on the front.*

13. Mary placed the box on a table, so that the two longer dimensions were on the table and height of the box was perpendicular to the table's surface.

"The angle between the diagonal of the box and the surface of the table," she said, "—I'll call that α."

The angle α contains the diagonal of the box and a line in the plane of the tabletop and it sits entirely in a plane perpendicular to the tabletop. What is the tangent of α? *Be sure to simplify your answer.*

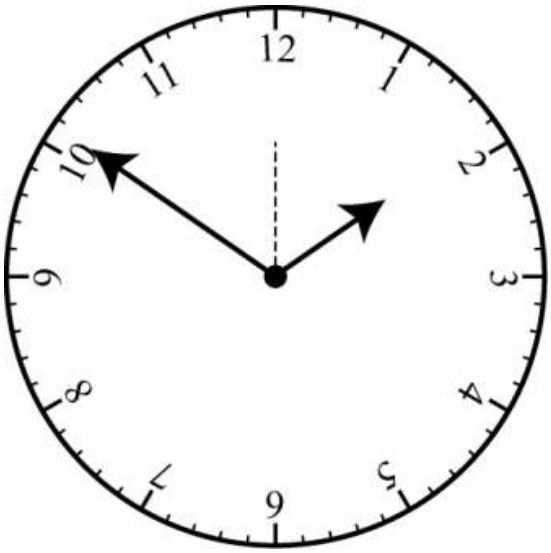
14. Inside Mary's box was a perfect ellipsoid. (An ellipsoid may be thought of as a sphere that has been elongated by varying factors along its three perpendicular axes.) The ellipsoid was tangent to all the sides of the box, and the axes of the ellipsoid were perpendicular to the sides of the box. What was the volume of the ellipsoid?



— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 11:.	**Answer to Problem 12:**	**Answer to Problem 13:**	**Answer to Problem 14:**
cubic inches	inches	tan α =	cubic inches

A conventional clock has two hands. The shorter hand, called the hour-hand, goes a full 360° around the clock face in 12 hours; the longer hand, called the minute-hand, goes a full 360° in 60 minutes. In the figure is shown a clock at a special time of day when the angle that the hour hand makes with the vertical is exactly equal to the angle the vertical makes with the minute hand.



The next several questions, of varying difficulty, discuss this situation. The next page is left blank as a place to put your work.

15. When the minute-hand of a clock sweeps 1° of arc, how many seconds elapse?

16. When the hour-hand of a clock sweeps 1° of arc, how many minutes elapse?

17. What is the angle of the hour-hand from the vertical in the figure above? *Your answer must be in degrees, and simplified per the terms of the front of this Competition.*

18. What is the time on the clock's face, rounded expertly, if needed, to the nearest second? *Answer in the form indicated. For example, 12 noon would be reported* 1 2 : 0 0 : 0 0 .

— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 15:.	**Answer to Problem 16:**	**Answer to Problem 17:**	**Answer to Problem 18:**
seconds	minutes	degrees	__ __ : __ __ : __ __

Acknowledgement: The last problem on this Competition is by Sam Loyd.