THE 31ST PEE – DEE REGIONAL HIGH – SCHOOL
MATHEMATICS TOURNAMENT

# *Written Competition*

SPONSORED BY FRANCIS MARION UNIVERSITY
AND THE PEE DEE EDUCATION CENTER

TUESDAY 2007 DECEMBER 04

## Instructions

- Do not turn over this page until instructed to do so.
- Neatly print (not sign) your name *as you wish it to appear if you are given an award.*
- During the examination, no calculators are allowed.
- Answers with fractions should be reported in lowest form. Answers involving $\pi$ should be written as such (that is, do *not* use 3.14 as an approximation to $\pi$), and similarly for answers with square-roots.
- Place your final answers in the boxes on each page designated for that purpose. If you change your final answer, cross out the box and label where your final answer is clearly and emphatically. Answers not in a box will not be scored, and answers in a wrong box are marked wrong.

Name (print neatly and fully):

________________________________________________________

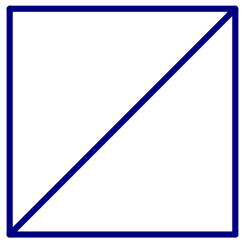
High School: _______________________________________________

*— Do not write in these spaces. —*

| | | | | |
|---|---|---|---|---|
| Page **1**. # 1, 2, 3 | Page **2**. # 4, 5, 6 | | **Total Correct** | **Weighted Sum*** |
| Page **3**. # 7, 8, 9 | Page **4**. # 10, 11, 12 | **Awards** | | |
| Page **5**. #13, 14. 15 | Page **6**. #16, 17, 18 | | | |

* Used only in tie-breaking.

**1.** A square has an area of 3 square units. What is the length of its diameter?



| **Answer to Problem 1:** |
|---|
| |

**2.** Simplify and express with the denominator rationalized: $\frac{\sqrt{5}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}}$

| **Answer to Problem 2:** |
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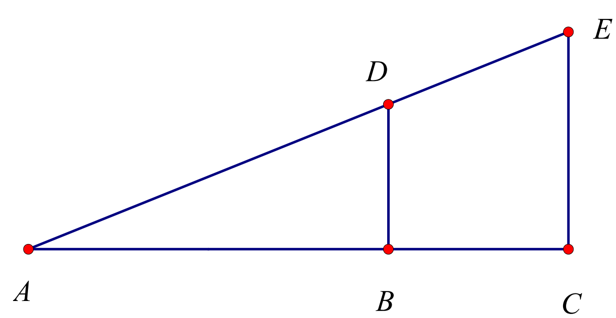
**3.** In the diagram, segment *AB* is twice as long as is segment *BC*. What is the ratio of segment *BD* to segment *CE*?




| **Answer to Problem 3:** |
|---|
| $\frac{\overline{BD}}{\overline{CE}} =$ |

**4.** The number $102210121_3$ is written in base 3. Write this same number in base 9.

| **Answer to Problem 4:** |
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**5.** Mary added all the numbers from 1 to 1000. What was her final sum?

| **Answer to Problem 5:** |
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**6.** The last two digits of the number 257834 are 34. What are the last two digits of $257834^2$?

| **Answer to Problem 6:** |
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**7.** If three people can move a quarter of a pile of sand in five hours, how long will it take four people to move two-thirds of a pile of sand? To gain credit for this problem, you must also express your answer in hours and minutes.

| **Answer to Problem 7:** |
|---|
| hours + minutes |

**8.** The sum of two real numbers is 8. The sum of the *squares* of the two numbers is 28. What is the product of the two numbers?

| **Answer to Problem 8:** |
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**9.** Johnny rolled three dice. The dice are regular, cubic dice numbered from 1 to 6. What is the probability that the numbers on the three dice added to 10?

| **Answer to Problem 9:** |
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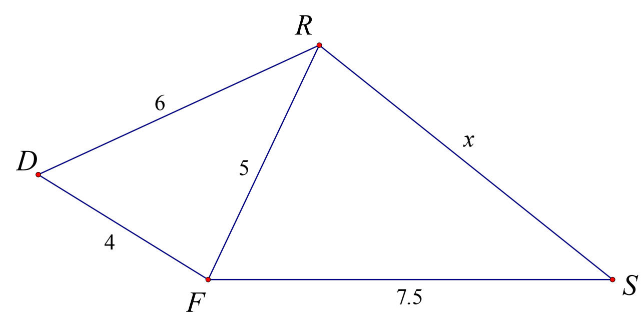
**10.** Find those real numbers $A$, $B$, and $C$ so that the following equation is an identity in $x$:

$$\frac{4x^2+9x-1}{x^3+2x^2-x-2} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+2}$$

| **Answer to Problem 10:** |
|---|
| $A$ = $B$ = $C$ = |

**11.** In the diagram, $\angle RFS = \angle RDF$. As marked, $\overline{FD} = 4$ units, $\overline{FR} = 5$ units, and $\overline{FS} = 7.5$ units. What is the length of $\overline{RS}$?




| **Answer to Problem 11:** |
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**12.** Simplify: $\log_{\sqrt{2}}(\sqrt[5]{4})$.

| **Answer to Problem 12:** |
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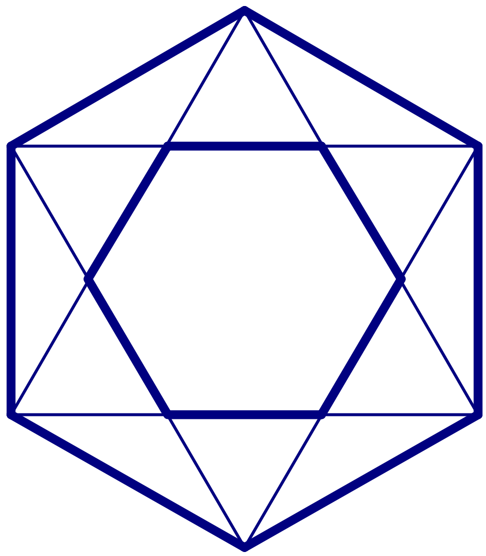
**13.** When alternate vertices of a regular hexagon are joined, another, smaller hexagon appears in the interior of the resulting six-pointed star. What is the ratio of the area of the smaller, resulting hexagon to the area of the larger, original hexagon?



| **Answer to Problem 13:** |
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| |

**14.** Solve for $x$: $4^{2x+3}\,8^{3x-2} = 128$.

| **Answer to Problem 14:** |
| --- |
| $x =$ |

**15.** A sand timer is in the shape of a two perfect cones, except for a negligibly small aperture through which sand falls in a perfectly uniform rate from the upper chamber into the lower chamber. At the start, the height of the sand in the upper chamber was 4 cm above the aperture. After 30 minutes, the sand in the upper chamber reached to a height of 2 cm. In how many more minutes will there be no sand left in the upper chamber?

| **Answer to Problem 15:** |
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*Work for problems on this page may be written on the bottom of this page or on the next page or anywhere else on the competition. Be sure to copy your final answers into the answer blocks, because work appearing elsewhere on the competition will not be scored or judged.*

**16.** One may factor the sixth-degree polynomial $x^6 - 1$ in two different ways.

One way is to factor it first as the difference of two squares, and then factor the resulting sum and difference of two cubes:

$$\begin{aligned} x^6 - 1 &= (x^3)^2 - 1^2 \\ &= (x^3 + 1) \cdot (x^3 - 1) \\ &= (x + 1) \cdot (x^2 - x + 1) \cdot (x - 1) \cdot (x^2 + x + 1) \end{aligned}$$

The other way is to parse the original $x^6 - 1$ as the difference of two cubes, and then factor the resulting difference of two squares:

$$\begin{aligned} x^6 - 1 &= (x^2)^3 - 1^3 \\ &= (x^2 - 1) \cdot (x^4 + x^2 + 1) \\ &= (x + 1) \cdot (x - 1) \cdot (x^4 + x^2 + 1) \end{aligned}$$

From this information, factor $x^4 + x^2 + 1$ as the product of two other polynomials.

| **Answer to Problem 16:** |
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**17.** Between 8:00 *a.m.* on one day and 8:00 *a.m.* on the next day, how many times does the minute hand cross the hour hand on a conventional clock?

| **Answer to Problem 17:** |
|---|
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**18.** Between 8:00 *a.m.* on one day and 8:00 *a.m.* on the next day, how many times does the second hand cross the hour hand?

| **Answer to Problem 18:** |
|---|
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