THE 32ND PEE – DEE REGIONAL HIGH – SCHOOL
MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED BY FRANCIS MARION UNIVERSITY
AND THE PEE DEE EDUCATION CENTER

TUESDAY · 2008 · DECEMBER · 02

Instructions

- Do not turn over this page until instructed to do so.
- Neatly print (not sign) your name *as you wish it to appear if you are given an award.*
- During the examination, no calculators are allowed.
- Answers with fractions should be reported in lowest form. Answers involving π should be written as such (that is, do *not* use 3.14 as an approximation to π), and similarly for answers with square-roots. If a radical appear in the denominator of your answer, you must also *rationalize the denominator* in order to receive credit.
- Place your final answers in the boxes on each page designated for that purpose. If you change your final answer, cross out the box and label where your final answer is clearly and emphatically. Answers not in a box will not be scored, and answers in a wrong box are marked wrong.

Name (print neatly and fully):

____

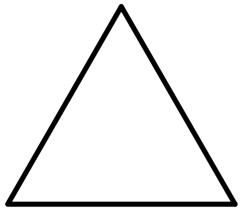
High School: ______________________________________

— Do not write in these spaces. —

Page **1**. # 1, 2, 3	Page **2**. # 4, 5, 6		**Total Correct**	**Weighted Sum***
Page **3**. # 7, 8, 9	Page **4**. # 10, 11, 12			
Page **5**. #13, 14. 15	Page **6**. #16, 17, 18	**Awards**		

* Used only in tie-breaking.

1. An equilateral triangle has each side being of length 1. What is its area?



Answer to Problem 1:

2. Simplify until the result is an integer over an integer and with all common factors canceled:

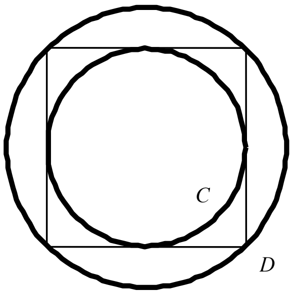
$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$

Answer to Problem 2:
$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} \ =$

3. Re-express with the denominator rationalized and perform whatever further simplifications that may occur: $\dfrac{4\sqrt{5}}{\sqrt{5}+1}$

Answer to Problem 3:
$\frac{4\sqrt{5}}{\sqrt{5}+1} \ =$

4. What is the ratio of the area of a circle *in*scribed in a square to the area of the circle *circum*scribed about the same square? That is, in the figure, what is the ratio of the area of circle C to the area of circle D?



Answer to Problem 4:

5. Mary has four children's blocks, each in the shape of a cube with the numerals "1" through "6" on the sides. Mary can arrange the blocks to form the number 3523 for instance, or use fewer blocks and make the numbers 641, 36, or 1. (The empty arrangement, using zero blocks, does *not* form a number at all.) How many different numbers can Mary form with her four children's blocks?

Answer to Problem 5:

6. A parabola $\{y = ax^2 + bx + c\}$ has its vertex at the point (–2, –24) and passes through the point (4, 3). What are the coefficients on x^2, x, and 1? Be sure that any fractions are in completely reduced form.

Answer to Problem 6:
$a =$ $b =$ $c =$

7. For positive numbers a and b, let

$$a \clubsuit b = \frac{a+b}{2} \quad \text{and} \quad a \heartsuit b = \sqrt{ab}.$$

Find and completely simplify $(a \heartsuit b) \clubsuit ((2a) \heartsuit (2b))$. You may express your answer either in traditional algebraic notation or with clubs and hearts.

Answer to Problem 7:

8. Recall that $n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$. Simplify $\log_{10}(1000!) - \log_{10}(999!)$.

Answer to Problem 8:

9. Two steps forward and one step back! Evaluate the following quantity. The pattern is perfectly preserved in the middle:

$1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + 10 + 11 - 12 + 13 + 14 - 15 + 16 + 17 - 18 + 19 + 20 - 21 + 22 + 23 - 24 + 25 + 26 - 27 + 28 + 29 - 30 + 31 + 32 - 33 + 34 + 35 - 36 + 37 + 38 - 39 + 40 + 41 - 42 + 43 + 44 - 45 + 46 + 47 - 48 + 49 + 50 - 51 + 52 + 53 - 54 + 55 + 56 - 57 + 58 + 59 - 60 + 61 + 62 - 63 + 64 + 65 - 66 + 67 + 68 - 69 + 70 + 71 - 72 + 73 + 74 - 75 + 76 + 77 - 78 + 79 + 80 - 81 + 82 + 83 - 84 + 85 + 86 - 87 + 88 + 89 - 90 + 91 + 92 - 93 + 94 + 95 - 96 + 97 + 98 - 99$

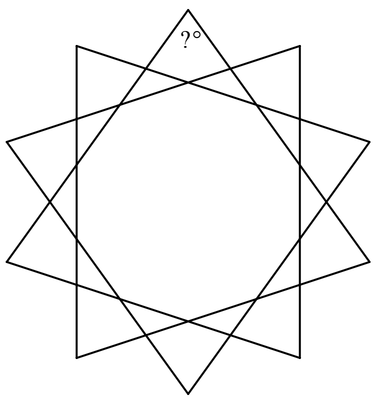
Answer to Problem 9:

10. Find those real numbers A, B, and C so that the following equation is an identity in x:

$$2x^2 + 3x - 7 \;=\; (Ax + B)(x + 5) + C$$

Answer to Problem 10:
A = B = C =

11. What is the angle at the tip of each outer vertex of the regular ten-pointed star shown below? You may answer in either radians or degrees.

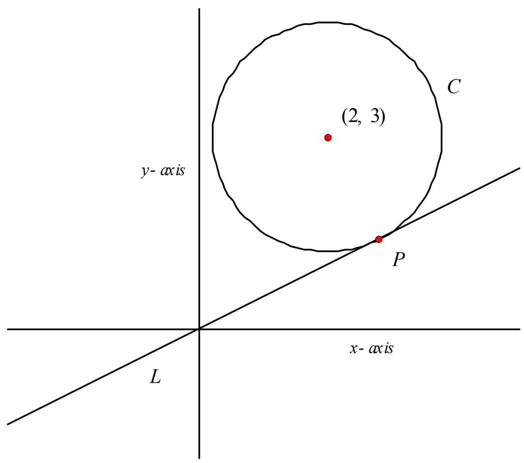


Answer to Problem 11:

12. Pipe A can fill a pool in 40 minutes. Pipe B can fill 2.5 pools in an hour. How long will it take Pipe A and Pipe B to fill 5 pools if they work together? (All pools of course are the same size.) To gain credit for this problem, you must also answer in hours and minutes.

Answer to Problem 12:
hr + min

13. Let L be the line $\{y = \frac{1}{2}x\}$. Let C be the circle centered at the point (2, 3) tangent to the line L. What are the coordinates of the point P, the point whereat the circle C is tangent to the line L? You may answer either with fractions reduced to lowest form or with decimals if you prefer.



Answer to Problem 13:
$P = (\qquad , \qquad)$

14. Find those real numbers A and B so that the following equation is an identity in θ:

$$\sin(4\theta) = A \sin\theta \cos^3\theta + B \sin^3\theta \cos\theta.$$

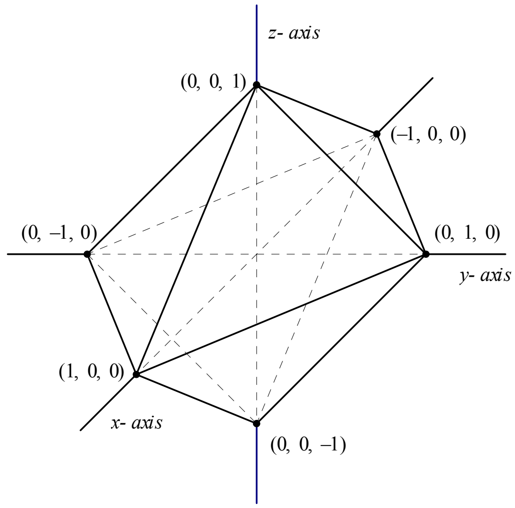
Answer to Problem 14:
$A =$ $\qquad$ $B =$

15. Solve for n: $1 + 2 + 3 + 4 + 5 + \cdots + (n-1) + n = 100n$.

In other words, solve for n: $\sum_{i=1}^{n} i = 100n$.

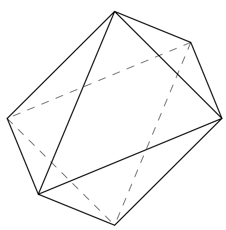
Answer to Problem 15:
$n =$

16. An octahedron is a regular solid bounded by eight congruent equilateral triangles. What is the volume of the octrahedron formed when the points (1, 0, 0), (0, 1, 0), (0, 0, 1), (–1, 0, 0), (0, –1, 0), and (0, 0, –1) are joined in the manner shown in the figure below?



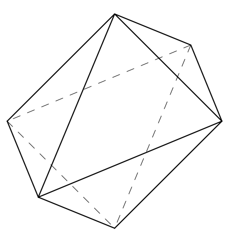
Answer to Problem 16:

17. What is the surface area of an octahedron each of whose edges is of length 1?



Answer to Problem 17:

18. What is the volume of an octrahedron each of whose edges is of length 1?



Answer to Problem 18:

***Note well:** Remember throughout this competition to rationalize the denominator if needed and to express any fractions in lowest form and to have answers in the proper boxes. The judges are instructed to reject answers that do not adhere to these protocols even if they are otherwise right.*