THE · 37TH · PEE – DEE · REGIONAL · HIGH – SCHOOL

MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED BY FRANCIS MARION UNIVERSITY
MU ALPHA THETA AND THE PEE DEE EDUCATION CENTER

TUESDAY · 2013 · DECEMBER · 03

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name *as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 250 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:	**Unacceptable**	**Acceptable**
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must “take out” all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$
All ratios must be written as a pure number in conventional notation. Translate “:” as “/” and, if necessary, simplify the resulting fraction.	1 : 2	$\frac{1}{2}$

↑ Name. (Print neatly and fully.)

↑ High School.

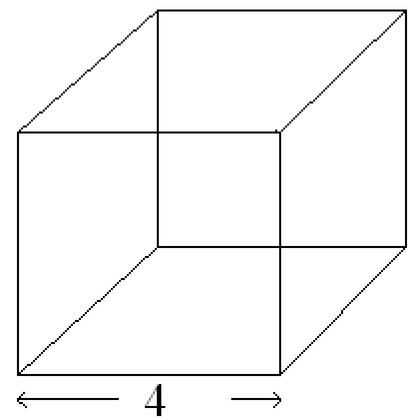
* Used only in tie-breaking.

—— For official use only ——

Page **1**. (# 1, 2, 3)	Page **2**. (# 4, 5, 6)
Page **3**. (# 7, 8, 9)	Page **4**. (# 10, 11, 12)
Page **5**. (# 13, 14, 15)	Page **6**. (# 16, 17, 18)
Total Correct	**Weighted Sum***

Awards

1. Each edge of a cube is 4 units in length. What is the volume of the cube?



2. Suppose $a_1 = \frac{1}{2}$, $a_2 = \frac{2}{3}$, $a_3 = \frac{3}{4}$, $a_4 = \frac{4}{5}$, and $a_5 = \frac{5}{6}$. If this pattern is maintained, for what value of n will a_n be equal to 0.95?

3. Mary has three octohedral dice. Each die has eight sides, and the sides are numbered 1 through 8. Mary could form the number 622, for example, from her dice. In all, how many different numbers can Mary form with her dice? *Be sure to calculate or simplify your answer per the instructions on the front.*

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**
cubic units	$n =$	

For your convenience, the following factorizations are provided. They may be useful in solving some or all of the problems on this page.

$$640 = 2^7 \cdot 5^1$$
$$43{,}560 = 2^3 \cdot 3^2 \cdot 5^1 \cdot 11^2$$

4. There are 43,560 square feet in an acre of land. A certain field is in the shape of a square, and contains exactly 10 acres of land. How many square feet are in the field?

5. What is the length of the side of the field in Problem 4? Answer in feet, *fully simplified.*

6. There are 640 acres in a square mile. How many square feet are in a square mile? *You must answer in factored form, by filling in the blanks for the exponents in the form below.*

— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 4: *Do not answer in factored form.*	**Answer to Problem 5:** *Do not answer in factored form.*	**Answer to Problem 6:** *You must answer in factored form.*
square feet	feet	$2^{\_\_} \cdot 3^{\_\_} \cdot 5^{\_\_} \cdot 11^{\_\_}$ square feet

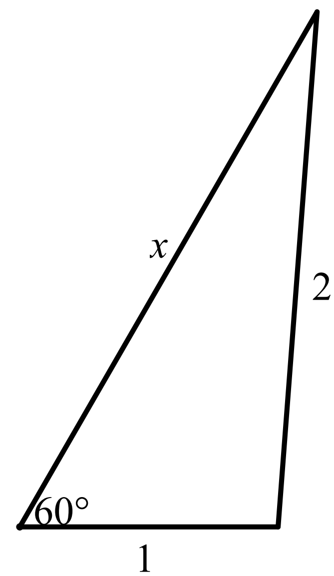
7. "Look!" said DaisyBelle, "we're sixty miles from Florence! At a mile per minute, we'll be there in an hour."

"Sooner than that!" said Studman. "We're going seventy miles per hour, not sixty."

To the nearest whole minute (do not include fractions), in how many minutes will Studman and DaisyBelle be in Florence?

8. Studman and DaisyBelle are planning a trip. They will travel 50 miles on the interstate, at a speed of 70 miles per hour, and then travel 50 miles along a mountain road, at 30 miles per hour. What will be the average speed of the entire trip? (It is *not* 50 m.p.h.)

9. The roof of an A-framed house is inclined 60° from the horizontal. A brace 2 meters long joins the roof and meets the gound 1 meter from where the roof meets the ground. In all, solve for x in the diagram. (*Note: Only one value for x is possible; do not answer two or more values for x.*)



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Answer to Problem 7: *Round to the nearest whole number.*	**Answer to Problem 8:** *Simplify as needed.*	**Answer to Problem 9:** *Answer exactly & simplify as instructed on the front.*
minutes	miles per hour	$x =$ meters

Throughout the problems on this page, $i = \sqrt{-1}$. Also, all factorization are presumed to be nontrivial. For example, no credit will be awarded for saying that $x^2 + 4 = 1 \cdot (x^2 + 4)$.

10. Notice $i^2 = -1$. Factor $x^2 + 4$ as the product of two polynomials with complex coefficients.

11. A Gaussian integer is by definition a complex number of the form $a + bi$, where both a and b are integers, such as $8 - 7i$ or $3 + 0i$. Factor 29 as the product of two Gaussian integers.

12. Factor the cubic polynomial $x^3 + 8i$ as the product of two polynomials with complex coefficients.

— In order to receive credit, answers must appear in these boxes. —

Answer to Problem 10:	Answer to Problem 11:	Answer to Problem 12:

For the problems on this page, let

$$f(x) = x^2 - 1$$

and

$$g(x) = x^2 + 2x + 1$$

as functions of x. As usual, the notation $\circ$ denotes functional composition. Notice also in Problem 14 that the input to the elaborate function is x^2, not x.

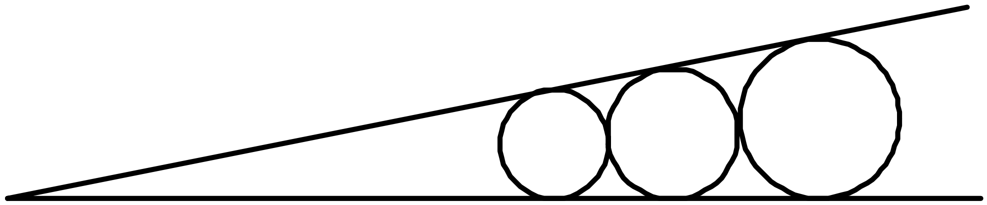
13. Find and simplify $(g \circ f)(x)$.

14. Find and simplify $(g \circ f \circ g \circ f \circ g \circ f)(x^2)$.

15. Find and simplify $(f \circ g \circ f \circ g \circ f \circ g \circ f \circ g \circ f)(x)$.

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 13:	**Answer to Problem 14:**	**Answer to Problem 15:**



16. In the diagram above, three circles are placed inside an angle as shown. Each circle is tangent to each of the two rays of the angle and is tangent also to its neighboring circle or circles. The smallest circle has area 2 square units, and the middle circle has area 3 square units. What is the area of the largest circle?

17. Let α be the original angle in which the circles were inscribed. From the data of Problem 16, the angle α is determined. What, in fact, is the sine of half of α?

$$\sin(\alpha / 2) = ?$$

18. A cone has three spheres placed inside it. Each sphere is tangent to the sides of the cone and is tangent also to its neighboring sphere or spheres. If the smallest sphere has volume 2 cubic units, and the middle sphere has volume 3 cubic units, what is the volume of the largest sphere?

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 16:	**Answer to Problem 17:**	**Answer to Problem 18:**
square units		cubic units