THE · 39TH · PEE – DEE · REGIONAL · HIGH – SCHOOL
MATHEMATICS · TOURNAMENT

# *Written Competition*

SPONSORED · BY · FRANCIS · MARION · UNIVERSITY
MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER
TUESDAY · 2015 · DECEMBER · 08

## Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name
*as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

*Because the judges must score over 300 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:*

| | **Unacceptable** | **Acceptable** |
|---|---|---|
| One must perform all arithmetic that evaluates to an integer. | $2^2 \cdot 3^3 \cdot 5$ | 540 |
| One must cancel all common factors in fractions of two integers. | 4/6 | 2/3 |
| In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part. | $2+\frac{5}{3}$ | $\frac{11}{3}$ or $3+\frac{2}{3}$ |
| In writing square-roots, one must "take out" all perfect squares. | $\sqrt{24}$ | $2\sqrt{6}$ |
| One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors. | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$ |
| | $\frac{2}{\sqrt{7}-1}$ | $\frac{\sqrt{7}+1}{3}$ |
| All ratios must be written as a pure number in conventional notation. Translate ":" as "/" and, if necessary, simplify the resulting fraction. | 1 : 2 | $\frac{1}{2}$ |

↑ Name. (Print neatly and fully.)

↑ High School. * Used only in tie-breaking.

Awards

—— *For official use only* ——

| | |
|---|---|
| Page **1**. (# 1, 2, 3) | Page **2**. (# 4, 5, 6) |
| Pages **3** and **4**. (# 7, 8, 9, 10, 11, 12) | |
| Page **5**. (# 13,14,15,16) | Page **6**. (# 17,18,19,20) |
| **Total Correct** | **Weighted Sum*** |

For the problems on this page, use the facts that there are sixty seconds in a minute, sixty minutes in an hour, twenty-four hours in a day, and three hundred and sixty-five days in a year. (That is, ignore leap-years.) Also, the speed of light is three hundred million meters per second, and a light-year is the distance light travels in one year.

**1.** How many seconds are in an hour?

**2.** How many hours are in a year?

**3.** The star Epsilon Eridani is a fairly faint star to the east of Rigel and is one of the closest stars to our solar system. It is 10 light-years distant from us. How far is this star from us, when measured in meters? To receive credit for this problem, you must

- answer in meters, as is pre-printed, and
- answer to three significant digits of accuracy and in scientific notation, for example $1.23 \times 10^{45}$. The form in the answer box should help you out: put an Arabic numeral in each underlined spot.

*— In order to receive credit, answers must appear in these boxes and be in the form specified. —*

| **Answer to Problem 1:** | **Answer to Problem 2:** | **Answer to Problem 3:** |
|---|---|---|
| An hour contains<br><br>seconds | A year contains<br><br>hours | The distance from the here to Epsilon Eridani is<br>$\_\_ . \_\_ \_\_ \times 10^{\_\,\_}$<br>meters |

**4.** At exactly 1:37 *p.m.*, Daisy-Belle departed from the city of Florence, South Carolina traveling west on I-20 going 60 miles per hour. At 2:07 *p.m.*, Studman also departed from Florence, traveling west on I-20, going 70 miles per hour. Assume perfect traveling conditions. At what time will Studman and Daisy-Belle be at the same place on I-20? *Answer to the nearest minute, for example " 0 4 : 1 5 p.m. ".*

**5.** As further answer to the problem above, where will Daisy-Belle and Studman be when they come to the same place on I-20?

**6.** When was the distance between Florence and Studman equal to twice the distance between Studman and Daisy-Belle? *Answer as a time of day to the nearest minute, as in Problem 4.*

*— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —*

| **Answer to Problem 4:.** | **Answer to Problem 5:** | **Answer to Problem 6:** |
|---|---|---|
| They will meet at<br><br>__ __ : __ __ *p.m.* | They will meet<br><br>miles west of Florence | (Florence to Studman) was twice<br>(Studman to Daisy-Belle) at<br><br>__ __ : __ __ *p.m.* |

**7.** **8.** **9.** **10.** **11.** **12.**

All logarithms on this page and the next are in base ten: $\log x = \log_{10}(x)$.

In the year 1624, Henry Briggs, continuing the work of John Napier, published a table of logarithms. This problem recreates a small (very small) part of what Mr. Briggs did to construct that table. Mr. Briggs knew all the properties of logarithms, and he also computed the quantities shown to the right. For your convenience, the values of several integral powers of 2 are also provided. All calculations are carried to five decimal places beyond the decimal point.

$$\log 2 = 0.30103$$
$$\sqrt{2} = 1.41421$$
$$\sqrt{10} = 3.16228$$
$$\sqrt{3.16228} = 1.77828$$
$$\sqrt{1.77828} = 1.33352$$
$$\log 10 = 1.00000$$

$$2^{-3} = 0.12500$$
$$2^{-2} = 0.25000$$
$$2^{-1} = 0.50000$$
$$2^{0} = 1$$
$$2^{1} = 2$$
$$2^{2} = 4$$
$$2^{3} = 8$$
$$2^{4} = 16$$
$$2^{5} = 32$$
$$2^{6} = 64$$
$$2^{7} = 128$$

Mr. Briggs took the information shown, together with the properties of logarithms, and derived the values of several logarithms. He then ordered his answers into a table. Your task is to do the same. Figure out as many other values for logarithms as you can, and then place your findings into the table below. Your answers should be carried to five decimal places beyond the decimal point. For each value you can figure out, you will receive 1/3 point of credit; quite something since there are only 20 points on this entire competition.

To help you out even more, we call your attention to these facts:

$2 \times 3.16228 = 6.32456$ $\quad$ $2 \times 1.77828 = 3.55656$ $\quad$ $2 \times 1.33352 = 2.66704$

Ready? Set? Then go figure out the values of some logarithms! And then place your determinations into the proper spaces in the answer table. The next page is left blank for you to place your work. *Lenience will be granted in how you choose to round the last digit of each answer.*

*Answer by filling in spots with Arabic numerals.*

| **Answers to Problems 7, 8, 9, 10, 11, and 12, collectively** | | | |
|---|---|---|---|
| log 1.00000 = | ___ . ___ ___ ___ ___ ___ | log 2.66704 = | ___ . ___ ___ ___ ___ ___ |
| log 1.25000 = | ___ . ___ ___ ___ ___ ___ | log 3.16228 = | ___ . ___ ___ ___ ___ ___ |
| log 1.28000 = | ___ . ___ ___ ___ ___ ___ | log 3.20000 = | ___ . ___ ___ ___ ___ ___ |
| log 1.33352 = | ___ . ___ ___ ___ ___ ___ | log 3.55656 = | ___ . ___ ___ ___ ___ ___ |
| log 1.41421 = | ___ . ___ ___ ___ ___ ___ | log 4.00000 = | ___ . ___ ___ ___ ___ ___ |
| log 1.60000 = | ___ . ___ ___ ___ ___ ___ | log 5.00000 = | ___ . ___ ___ ___ ___ ___ |
| log 1.77828 = | ___ . ___ ___ ___ ___ ___ | log 6.32456 = | ___ . ___ ___ ___ ___ ___ |
| log 2.00000 = | ___ . ___ ___ ___ ___ ___ | log 6.40000 = | ___ . ___ ___ ___ ___ ___ |
| log 2.50000 = | ___ . ___ ___ ___ ___ ___ | log 8.00000 = | ___ . ___ ___ ___ ___ ___ |

*— This competition continues for two more pages. —*

On this page, all angles are measured in radians. Recall that sine and cosine are a functions that can take any real number as input, but return only numbers between –1 and +1 as output. Arcsine, however, takes only numbers between –1 and +1 as input, and returns only numbers between $-\pi/2$ and $+\pi/2$ as output. Also, arccosine is a function that takes only numbers between –1 and +1 as input, and returns only numbers between 0 and $\pi$ as output.

**13.** Evaluate: $\arcsin(\sin(23\pi/4))$. The answer is *not* $23\pi/4$.

**14.** Evaluate: $\sin(\arccos(\arcsin(\cos(3\pi/2))))$. The answer is *not* $3\pi/2$.

**15.** Evaluate: $\arccos(\arcsin(\sin(\cos(5\pi/3))))$. The answer is *not* $5\pi/3$.

**16.** Evaluate: $\arcsin(\sin(\tan(\pi/3)))$.

*— In order to receive credit, answers must appear in these boxes and be properly simplified. —*

| **Answer to Problem 13:** | **Answer to Problem 14:** | **Answer to Problem 15:** | **Answer to Problem 16:** |
|---|---|---|---|
| | | | |

NOTE WELL: *Only those students who get Problem 17 correct are eligible to receive credit for Problems 18, 19, and 20.*

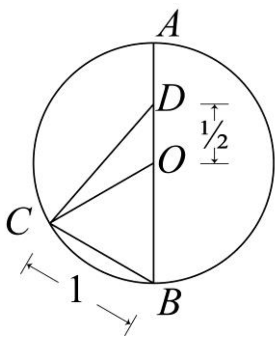
In the problems on this page, points $A$, $B$, and $C$ lie on a circle of radius 1 centered on the point $O$. The points $A$, $D$, $O$, and $B$ are collinear with one another.

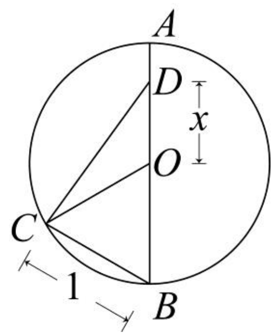
**17.** Given that $OA = OB = OC = 1$, and given that $OD = 1/2$ and $BC = 1$, find $CD$. *Be careful that no trigonometric or arctrigonometric functions appear in your final answer and to simplify the radical if needed.*

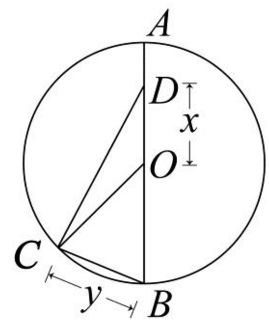
**18.** In the same diagram as for the last problem, what is the measure of $\angle OCA$? (You may wish to draw $CA$ yourself.) *You must answer in radians, as indicated in the answer box.*

**19.** Given (again) that $OA = OB = OC = 1$, and $OD = x$ and $BC = 1$, find $CD$ as a function of $x$. *Be careful that no trigonometric or arctrigonometric functions appear in your answer.*

**20.** Given (yet again) that $OA = OB = OC = 1$, $OD = x$, and $BC = y$, find $CD$ as a function of $x$ and $y$. *Be careful that no trigonometric or arctrigonometric functions appear in your answer.*




*— In order to receive credit, answers must appear in these boxes and be in the form specified. —*

| **Answer to Problem 17:** | **Answer to Problem 18:** | **Answer to Problem 19:** | **Answer to Problem 20:** |
|---|---|---|---|
| | radians | | |

**Answer to Ultimate Tiebreaker:**