

<b>Name of Program/Department:</b>	Mathematics Program
<b>Year:</b>	2021-22
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### ***Program Mission Statement***

A primary purpose of the Department of Mathematics at Francis Marion University is to offer all University students a varied and well-balanced curriculum of undergraduate education in mathematics. In the liberal-arts tradition, the courses in the curriculum teach students to think logically, to analyze problems and solve them appropriately, and to communicate their ideas clearly.

The department also provides a broad range of entry-level courses in order to meet the needs of students with widely varying mathematical backgrounds and to provide them with skills appropriate for their selected majors. The mathematics courses that satisfy the General Education requirement in mathematics are designed to help students achieve *Goal 4: The ability to use fundamental mathematical skills and principles in various applications.*

Equally important, the curriculum provided by the Department leads to baccalaureate degrees in two distinct but overlapping areas: mathematical sciences and teacher licensure in mathematics. These courses prepare students for careers in education, business, industry, and government. They also prepare those students of sufficient interest and ability for further study of mathematics at the graduate level.

### ***Program Learning Outcomes***

1. Students should be able to analyze problems involving various applications and solve them using appropriate mathematical skills, principles, and technology.
2. Students should be able to present oral and written solutions in a structured format that can be understood by a general audience.
3. Students should recognize and appreciate the applicability, beauty, and power of mathematics.
4. Students should be confident in their abilities to use mathematics to solve various problems.

## *Executive Summary*

The Department of Mathematics uses several direct and indirect assessments. The direct assessments of Student Learning Outcomes (SLO) 1.0 (Outcomes 1-4), SLO 2.0 (Outcomes 1-2), SLO 3.0 (Outcomes 1-2), and SLO 5.0 (Outcomes 1-2) are evaluated using a calculus performance rubric, an elementary proof performance rubric, a technology usage performance rubric, and a communication performance rubric from student work samples. Values are the percentages of students who met or exceeded faculty expectations based on the rubrics. The indirect assessments of SLO 1.5<sup>†</sup>, SLO 2.3, SLO 3.3, SLO 4.1-4.2, and SLO 5.4 are tabulated from student attitude surveys. Values are percentages of students who state that they are confident in their skills and abilities or have an appreciation for the beauty of mathematics as a singular discipline and its applications.

Academic year 2021-22 assessments show targets were achieved in 11 of 17 outcomes. SLO 1.4, which measures competence to calculate and use gradients and partial derivatives, achieved its new target of 80%. SLO 2.1, which measured students' ability to read elementary proofs, increased for the fourth consecutive year and achieved its target. SLO 3.2, which measures students' ability to write computer programs, has had volatile scores but achieved its target. All students felt confident in mathematical abilities for SLO 1.5, SLO 2.3, and SLO 3.3. Students continue to meet targets in the appreciation of the beauty of mathematics as a singular discipline and its applications (SLO 4.0). One student response out of nine students for SLO 5.4, which measures students' confidence in communicating mathematics in written form and oral presentations, caused SLO 5.0, the ability to effectively communicate mathematics in written form and oral presentations, to fall short a perfect score.

Targets were not achieved in 6 of 17 outcomes. Assessment problems in SLO 1.1 and 1.3 were revised prior to academic year 2019-20. SLO 1.1 – SLO 1.3 values were mixed and did not achieve the targets. Further analysis is needed to see if the revisions are favorable. SLO 2.2 value, which measures students' ability to write elementary proofs, decreased significantly and did not achieve its target. The instructors of Math 230 Discrete Mathematics feel that some of the mathematical proofs required in SLO 2.2 don't align well with the problems assigned in the course textbook Discrete Mathematics by Kenneth Rosen. The instructors of Math 230 Discrete Mathematics and Math 311 Transitions to Higher Mathematics will revise problems to more closely align with mathematical proof problems in textbooks for Math 230 and Math 311. SLO 3.1 value, which measures students' ability to read computer programs, increased from last year but did not achieve the target. As previously mentioned SLO 5.4 did not achieve its target due to the response of one student out of nine student responses.

While the 2021-22 Academic Calendar was not adjusted due to COVID-19, masking, social-distancing, and absences posed challenges for instructional methods and assessment instruments. Looking at historical data, the 2021-22 assessment values are favorable compared to the 4-year baseline, except for SLO 2.2.

<sup>†</sup> SLO 1.5 is an abbreviated notation for SLO 1.0 Outcome 5.

## *Student Learning Outcomes*

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Students will demonstrate competence to calculate limits and derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499).

Outcome 2: Students will demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499).

Outcome 3: Students will demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499).

Outcome 4: Students will demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499).

Outcome 5: Students will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence (Math 499).

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Students will be able to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311).

Outcome 2: Students will be able to write elementary proofs (Math 230/311).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs (Math 499).

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Students will be able to read computer programs that model various mathematical applications (Math 213).

Outcome 2: Students will be able to write computer programs that model various mathematical applications (Math 213).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems (Math 499).

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Students will respond to a statement concerning their appreciation for the beauty of mathematics as a singular discipline (Math 499).

Outcome 2: Students will respond to a statement concerning their understanding of the importance of mathematics in real world applications (Math 499).

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Students will communicate mathematics in a written presentation (Math 499).

Outcome 2: Students will communicate mathematics in an oral presentation (Math 499).

Outcome 3: Secondary education students will demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching).

Outcome 4: Students will respond to a statement concerning their confidence in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499).

### *Assessment Methods*

The direct assessments (SLO 1.1-1.4, SLO 2.1-2.2, SLO 3.1.3.2, and SLO 5.1-5.2) are evaluations of student work samples normally part of the end of course exam using performance rubrics for calculus, elementary proof, technology usage, and communication. The indirect assessments (SLO 1.5, SLO 2.3, SLO 3.3, SLO 4.1-4.2, and SLO 5.4) are tabulated from a senior survey administered in Math 499 at the end of the semester.

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

For outcomes 1-4, instructors of Calculus sequence courses (Math 201, 202, 203, 306) and Mathematics Capstone Course (Math 499) will collect samples of student solutions to problems or other work that call for students to demonstrate proficiency of basic computational techniques in the calculus sequence. Student solutions will be evaluated based on a calculus performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target for outcomes 1-3 is for 70% of students to meet or exceed faculty expectations. The target for outcome 4 is for 80% of students to meet or exceed faculty expectations. For outcome 5, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

For outcomes 1-2, instructors of Discrete Mathematics I (Math 230) and Transition to Higher Mathematics (Math 311) will collect samples of student solutions or relevant problems of other work to demonstrate the ability to understand and construct elementary proofs. Student solutions will be evaluated based on a proof performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target is for 70% of students to meet or exceed faculty expectations. For outcome 3, students will complete a senior survey in the

Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

For outcomes 1-2, instructors of Scientific Programming in Python (Math 213) will collect samples of student solutions to relevant problems or other work to demonstrate the ability to use appropriate technology to solve mathematical problems. Student solutions will be evaluated based on a programming performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target is for 70% of students to meet or exceed faculty expectations. For outcome 3, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Students will complete senior surveys in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree to statements concerning their appreciation for the beauty of mathematics and their understanding of the importance of mathematics. The target is for 95% of students to agree or strongly agree.

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

For outcomes 1-3, instructors of the Mathematics Capstone Course (Math 499) and supervisors of student teachers will collect samples of student work and will attend presentations that call for students to effectively communicate mathematics. Student work and presentations will be evaluated based on a communication performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target for outcomes 1-3 is for 80% of students to meet or exceed faculty expectations. For outcome 4, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

## ***Assessment Results***

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Fifty-four of eighty-seven (62.1%) students did demonstrate competence to calculate limits and derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499). This benchmark was not achieved.

Outcome 2: Thirty-three of fifty-four (61.1%) students did demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499). This benchmark was not achieved.

Outcome 3: Thirty-seven of sixty-one (60.7%) students did demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499). This benchmark was not achieved.

Outcome 4: Thirty-one of thirty-four (91.2%) students did demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499). This benchmark was achieved.

Outcome 5: Nine of nine (100.0%) students did respond that they were confident in their computational techniques in the calculus course sequence (Math 499). This benchmark was achieved.

*SLO 1.1, SLO 1.2, and SLO 1.3 did not achieve the benchmark of 70%.*

*SLO 1.4 achieved the benchmark of 70%.*

*SLO 1.5 achieved the benchmark of 95%.*

*SLO 1.0's overall benchmark was not achieved.*

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Fifteen of twenty-one (71.4%) students did show ability to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311). This benchmark was achieved.

Outcome 2: Nine of twenty-one (42.9%) students did show ability to write elementary proofs (Math 230/311). This benchmark was not achieved.

Outcome 3: Nine of nine (100.0%) students did respond that they were confident in their ability to understand and construct elementary proofs (Math 499). This benchmark was achieved.

*SLO 2.1 did achieve the benchmark of 70%.*

*SLO 2.2 did not achieve the benchmark of 70%.*

*SLO 2.3 achieved the benchmark of 95%.*

*SLO 2.0's overall target was not achieved.*

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Thirteen of twenty (65.0%) students did show ability to read computer programs that model various mathematical applications (Math 213). This benchmark was not achieved.

Outcome 2: Fourteen of twenty (70.0%) students did show ability to write computer programs that model various mathematical applications (Math 213). This benchmark was achieved.

Outcome 3: Four of four (100.0%) students did respond that they were confident in their ability to use appropriate technology to solve mathematical problems (Math 499). This target was achieved.

*SLO 3.1 did not achieve the benchmark of 70%.*

*SLO 3.2 did achieve the benchmark of 70%.*

*SLO 3.3 did achieve the benchmark of 95%.*

*SLO 3.0's overall benchmark was not achieved.*

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Nine of nine (100.0%) students did respond that they had an appreciation for the beauty of mathematics as a singular discipline (Math 499). This benchmark was achieved.

Outcome 2: Nine of nine (100.0%) students did respond that they had an understanding of the importance of mathematics in real world applications (Math 499). This benchmark was achieved.

*SLO 4.0's overall benchmark was achieved.*

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Eleven of eleven (100.0%) students did communicate mathematics effectively in a written presentation (Math 499). This benchmark was achieved.

Outcome 2: Eleven of eleven (100.0%) students did communicate mathematics effectively in an oral presentation (Math 499). This benchmark was achieved.

Outcome 3: Two of two (100.0%) student did demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching). This benchmark was achieved.

Outcome 4: Eight of nine of (88.9%) students did respond that they were confident in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499). This benchmark was not achieved.

*SLO 5.1, SLO 5.2, and SLO 5.3 did achieve the benchmark of 90%.*

*SLO 5.4 did not achieve the benchmark of 95%.*

*SLO 5.0's overall benchmark was not achieved.*

**Table 1.0: Assessment Results**

Assessment	Four-year Baseline	2017-18	2018-19	2019-20	2020-21	2021-22
SLO 1.0 Outcome 1	52.1	46.1	47.9	80.0	72.5	62.1
Outcome 2	59.6	56.1	50.7	72.7	56.8	61.1
Outcome 3	60.9	51.2	67.4	76.9	64.7	60.7
Outcome 4	86.5	88.2	94.7	94.4	90.9	91.2
Outcome 5	100.0	100.0	100.0	100.0	100.0	100.0
SLO 2.0 Outcome 1	71.3	76.5	57.1	65.2	66.7**	71.4
Outcome 2	68.8	82.4	46.4	82.6	66.7	42.9
Outcome 3	100.0	100.0	100.0	100.0	100.0	100.0
SLO 3.0 Outcome 1	81.5	76.5	87.5	88.2	57.9	65.0
Outcome 2	74.2	70.6	87.5	64.7	94.7	70.0
Outcome 3	97.2	100.00	88.9	100.0	100.0	100.0
SLO 4.0 Outcome 1	100.0	100.0	100.0	100.0	100.0	100.0
Outcome 2	100.0	100.0	100.0	100.0	100.0	100.0
SLO 5.0 Outcome 1	89.1	85.7	88.9	100.0	100.0	100.0
Outcome 2	85.4	71.4	88.9	100.0	100.0	100.0
Outcome 3	100.0	*	100.0	100.0	100.0	100.0
Outcome 4	100.0	100.0	100.0	100.0	100.0	88.9

\* No students participated in student teaching during the academic year.

\*\* SLO 2.1 was not evaluated in Spring 2021.

### **Action Items**

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Students will demonstrate competence to calculate limits and derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499).

Outcome 2: Students will demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499).

Outcome 3: Students will demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499).

Outcome 4: Students will demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499).

Outcome 5: Students will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence (Math 499).

*The assessment problems of Outcomes 1 and 3 were revised prior to the 2019-20 academic year because instructors felt that the problems did not accurately measure student performance. The resulting percentages are mixed for 2021-22 data with SLO 1.1 and SLO 1.3 decreasing and SLO*



*1.2 increasing compared to 2020-21 data. We believe the new assessment problems should accurately measure student learning, so the targets will remain at the same levels.*

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Students will be able to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311).

Outcome 2: Students will be able to write elementary proofs (Math 230/311).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs (Math 499).

*Assessment scores for Outcome 1 increased again for the fourth consecutive year. Assessment score for Outcome 2 decreased significantly. The instructors of Math 230 Discrete Mathematics feel that some of the mathematical proofs included in SLO 2.2 do not align well with the problems assigned in the course textbook Discrete Mathematics by Kenneth Rosen. Instructors will revise problems to more closely align with mathematical proof problems in textbooks for Math 230 and Math 311. The targets will remain at the same levels since assessments will be revised.*

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Students will be able to read computer programs that model various mathematical applications (Math 213).

Outcome 2: Students will be able to write computer programs that model various mathematical applications (Math 213).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems (Math 499).

*The programming language used in the computer programming course was changed from FORTRAN to Python beginning in the academic year 2019-20. This required a course change by removing Math/CS 212 Introduction to FORTRAN and adding a new course, Math 213 Scientific Programming in Python. The resulting percentages are mixed for 2021-22 with SLO 3.1 increasing compared to 2020-21 data but not achieving the benchmark and SLO 3.2 decreasing but still achieving the benchmark. The targets will remain at the same levels.*

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Students will respond to a statement concerning their appreciation for the beauty of mathematics as a singular discipline (Math 499).

Outcome 2: Students will respond to a statement concerning their understanding of the importance of mathematics in real world applications (Math 499).

*These benchmarks were met and no action is planned.*

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Students will communicate mathematics in a written presentation (Math 499).

Outcome 2: Students will communicate mathematics in an oral presentation (Math 499).

Outcome 3: Secondary education students will demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching).

Outcome 4: Students will respond to a statement concerning their confidence in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499).

*Targets for Outcomes 1-3 were increased to 90% last year. Outcome 4 was less than 100% due to the response of one student. Targets will remain at the same level.*

## *Appendices*

### Goal 1 Outcome 1

#### Assessment Problem 1:

Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^3 + 5x^2 - 14x}$  (any indeterminate limit that requires algebraic manipulation is suitable)

Score	Criteria
1	Incorrect approach. Evaluated limit incorrectly as written.
2	Correct approach (either algebraic manipulation or L'Hospital's rule) but calculation mistake in algebra or arithmetic.
3	Correct limit using appropriate calculus.

#### Assessment Problems 2 and 3:

Two differentiation problems that require use of at least two of the following differentiation rules: chain rule, product rule and quotient rule. Examples of such problems are as follows:

$$\frac{x \cdot e^x}{\cos x}, \quad \frac{x \cdot \cot x}{7^x}, \quad \frac{\ln x^2}{x}, \quad (2x^3 + 4x^{-1} + 3)^{10} \cdot \log_3 x, \quad x^2 \cdot \arctan 5x, \quad \sin(\cos x^4)$$

#### Likert scale for Problems 2 and 3:

Score	Criteria
1	Incorrect approach and/or calculus errors.
2	Properly identified rules required for the solution; algebra, arithmetic and/or notation errors.
3	Correct answer with proper notation.

#### Assessment Problem 4:

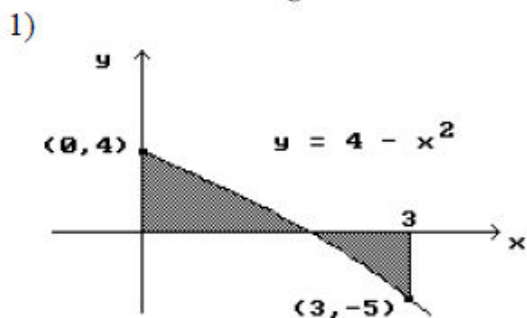
Use the Extreme Value Theorem to find the absolute maximum and absolute minimum of a cubic polynomial on a closed interval.

#### Likert scale for Problem 4:

Score	Criteria
1	Incorrect approach, incorrect derivative, critical values incorrect or incorrect mathematics used in finding them and/or no calculus used.
2	Correct critical values found. Found local maximum/minimums using critical points but does not check endpoints.

- 3 | Properly finds absolute maximum and minimum using EVT and correctly identifies critical numbers.

Find the area of the shaded region.



Total sum of 4-7 produces an overall score of 1, sum of 8-10 produces an overall score of 2, and sum of 11-12 produces an overall score of 3.

Goal 1 Outcome 2

Assessment Problems:

**Calculus Performance rubric for Outcome 2 for Problem 1:**

- 1 - Does not meet faculty expectations
  - \*Integral set up incorrectly.
  - \*Integral set up correctly but evaluated incorrectly at a conceptual level such as the incorrect use of the Evaluation Theorem or the absolute value of the integral is not taken.
- 2 - Meets faculty expectations
  - \*Integrals are set up properly.
  - \*Integrals are evaluated properly with possible minor calculation errors.
  - \*Absolute value is taken to reflect the area of the entire shaded region.
- 3 - Exceeds faculty expectations
  - \*Correct solution with proper work shown and no calculation errors.

Use the substitution formula to evaluate the integral.

$$2) \int_{-1}^0 \frac{2t}{(3+t^2)^3} dt$$

Evaluate the integral.

$$3) \int 5x \sin x dx$$

**Calculus Performance rubric for Outcome 2 for Problems 2 and 3:**

Score	Criteria
1	<ul style="list-style-type: none"> <li>• Incorrect approach</li> </ul>

	<ul style="list-style-type: none"> <li>• Major calculation errors</li> </ul>
2	<ul style="list-style-type: none"> <li>• Properly identified rules required for the solution</li> <li>• Minor calculation errors including algebra and notation</li> </ul>
3	<ul style="list-style-type: none"> <li>• Correct answer with proper notation</li> </ul>

**Total sum of 3-5 produces an overall score of 1, 6-7 produces an overall score of 2, and 8-9 produces an overall score of 3.**

**Goal 1 Outcome 3**

**Assessment Problems:**

1. Describe in your own words what it means for a series  $\sum a_n$  to converge. Does the convergence of the sequence  $\{a_n\}$  imply the convergence of the series  $\sum a_n$ ?

Score	Criteria
1	Answered both questions incorrectly.
2	Answered one question correctly.
3	Answered both questions correctly.

2. (Choose one) Determine whether the series converges or diverges. You can use whatever test that you prefer. However, you must show how you use the test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$$

$$\sum_{n=0}^{\infty} \frac{2^n}{3^n+1}$$

Score	Criteria
1	Incorrect approach or no work shown.
2	Correct approach but some algebraic/arithmetic errors or insufficient work shown.
3	Correct approach with the required work shown.

3. Determine the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

Score	Criteria
1	Incorrect approach or no work shown.
2	Correct approach but some algebraic/arithmetic errors or insufficient work shown.
3	Correct approach with the required work shown.

Score	Criteria
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<b>1</b>	Answered both parts incorrectly.
<b>2</b>	Answered one part correctly.
<b>3</b>	Answered both parts correctly.

4. Let  $f(x) = e^{3x}$ . Find the Taylor series for the function  $f$  centered at 0. List the first six terms of the series and then write the series using  $\sum$  notation.

**Total sum of 4-7 produces an overall of 1, 8-10 produces an overall score of 2, and 11-12 produces an overall score of 3.**

#### Goal 1 Outcome 4

##### Assessment Problem:

Let  $f \in \mathbb{R}^2 \rightarrow \mathbb{R}^1$  with  $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y + 2$ . Compute the following functions, numbers, vector, and equation by hand.

(a)  $\frac{\partial}{\partial x} f(x, y) =$

(b)  $eval_{(x,y)=(1,2)} \frac{\partial}{\partial x} f(x, y) =$

(c)  $\frac{\partial}{\partial y} f(x, y) =$

(d)  $eval_{(x,y)=(1,2)} \frac{\partial}{\partial y} f(x, y) =$

(e)  $\nabla f(1,2) =$

(f, g) Find the equation of the plane  $\Pi$  tangent to the surface  $\{z = f(x, y)\}$  through the point  $(1, 2, 21)$ .

##### Calculus Performance rubric for Outcome 4:

- 1 (Not meet expectation) Student cannot find partial derivatives and evaluate them at the point specified.
- 2 (Meets expectation) Student can find partial derivatives and evaluate them at the point specified.

3 (Exceeds expectation) Student goes on to find the gradient and equation of the tangent plane.

**Goal 1 Outcome 5**

How would you respond to the following statement: I am proficient in elementary computational techniques in the calculus.

*Disagree*      *Agree*      *Strongly Agree*

**Goal 2 Outcome 1****Assessment Problem:**

For each proposition stated, if the proof given is in fact a valid proof, identify the method of proof used (Direct, Indirect Contrapositive, Contradiction, or Induction). If there is an error in the proposed proof, find and describe the problem. An error could be any of the following: the use of a fallacy argument, an algebraic error, the misuse of a definition, an incomplete proof, and so on.

**Proposition 1.** *If  $x$  and  $y$  are integers which are either both even or both odd, then  $x - y$  is even.*

*Proof.* Let  $x$  and  $y$  be integers. We consider two cases, according to whether  $x$  and  $y$  are both even or both odd.

Case 1:  $x$  and  $y$  are both even. Let  $x = 6$  and  $y = 2$ , which are both even. Then  $x - y = 4$ , which is even.

Case 2:  $x$  and  $y$  are both odd. Let  $x = 7$  and  $y = 1$ , which are both odd. Then  $x - y = 6$ , which is even.  $\square$

**Proposition 2.** *If  $m$  is an even integer and  $n$  is an odd integer, then  $3m + 5n$  is odd.*

*Proof.* Let  $m$  be an even integer and let  $n$  be an odd integer. Then  $m = 2k$  and  $n = 2k + 1$ , for some  $k \in \mathbb{Z}$ . Therefore, we have

$$\begin{aligned} 3m + 5n &= 3(2k) + 5(2k + 1) \\ &= 6k + 10k + 5 \\ &= 16k + 5 \\ &= 2(8k + 2) + 1. \end{aligned}$$

Since  $8k + 2$  is an integer, it follows that  $3m + 5n$  is odd.  $\square$

**Proposition 3.** *For all  $x \in \mathbb{R}$ ,  $|x| = 2$  if and only if  $x^2 - 4 = 0$ .*

*Proof.* Case 1: Consider any  $x \in \mathbb{R}$  for which  $|x| = 2$ . Then  $|x|^2 = 2^2 = 4$ . As  $|x|^2 = x^2$  for any  $x \in \mathbb{R}$ , it follows that  $x^2 - 4 = 4 - 4 = 0$ .

Case 2: Consider any  $x \in \mathbb{R}$  for which  $x^2 - 4 \neq 0$ . Then  $x^2 \neq 4$ , and so  $\sqrt{x^2} \neq \sqrt{4}$ . This implies  $|x| \neq 2$ , and so this case follows by contrapositive.  $\square$



**Proposition 4.** For all  $x \in \mathbb{Z}$ , if  $x^2 + 2x$  is even then  $x$  is even.

*Proof.* Consider any  $x \in \mathbb{Z}$  such that  $x$  is odd. Then  $x = 2k + 1$  for some  $k \in \mathbb{Z}$ . It follows that

$$\begin{aligned} x^2 + 2x &= (2k + 1)^2 + 2(2k + 1) = 4k^2 + 4k + 1 + 4k + 2 \\ &= (4k^2 + 8k + 2) + 1 \\ &= 2(2k^2 + 4k + 1) + 1. \end{aligned}$$

Therefore,  $x^2 + 2x$  is an odd integer, and the proposition follows.  $\square$

**Proposition 5.** For all  $n \in \mathbb{N}$ ,  $\sum_{j=1}^n (2j - 1) = n^2$ .

*Proof.* If  $n = 1$ , then

$$\sum_{j=1}^1 (2j - 1) = 2(1) - 1 = 1 = 1^2 = n^2.$$

Thus, the statement is true when  $n = 1$ . Now suppose that the statement is true when  $n = k$  for any  $k \geq 1$ , and thus

$$\sum_{j=1}^k (2j - 1) = k^2. (*)$$

Then for  $n = k + 1$ , we have

$$\begin{aligned} \sum_{j=1}^n (2j - 1) &= \sum_{j=1}^{k+1} (2j - 1) = \sum_{j=1}^k (2j - 1) + \sum_{j=k+1}^{k+1} (2j - 1) \\ &= \sum_{j=1}^k (2j - 1) + [(2(k + 1) - 1)] \\ &= k^2 + [2(k + 1) - 1] \quad \text{by } (*) \text{ above} \\ &= k^2 + 2k + 1 = (k + 1)^2 = n^2. \end{aligned}$$

Thus, if the statement is true when  $n = k$  for any  $k \geq 1$ , the statement has also been shown to be true for  $n = k + 1$ . Therefore, the statement must be true for all  $n \in \mathbb{N}$ .  $\square$

**Proof Performance rubric for Outcome 1:**

## Outcome 1

- (1) 0-2 correctly identified proposition(s) does not meet faculty expectations.
- (2) 3-4 correctly identified propositions meets faculty expectations.
- (3) 5 correctly identified propositions exceeds faculty expectations.

**Goal 2 Outcome 2****Assessment Problem:**

Provide mathematically correct, properly written proofs for the given statements, using the indicated proof technique if so directed.

**Assessment Problem:** Provide mathematically correct, properly written proofs for the given statements.

- (1) Let  $X$  be a non-empty set, and let  $A$ ,  $B$ , and  $C$  be subsets of  $X$ . Prove that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .
- (2) For all  $x, y \in \mathbb{Z}$ , prove that  $x + 3y$  is even if and only if  $x$  and  $y$  are either both even or both odd.
- (3) Prove that, for every positive integer  $n$ ,

$$\sum_{j=1}^n 4j + 6 = 2n(n + 4).$$

- (4) Prove that, for any  $x \in \mathbb{R}$ , if  $x^2(x + 1)$  is irrational then  $x$  is irrational.

**Proof Performance rubric for Outcome 2:**

Problems 1, 3, and 4 count for 1 proof and Problem 2 counts for 2 proofs:

- (1) 0-2 proven correctly: does not meet faculty expectations.
- (2) 3-4 proven correctly: meets faculty expectations.
- (3) 5 proven correctly: exceeds faculty expectations.

**Goal 2 Outcome 3**

How would you respond to the following statement: My ability to understand and construct elementary proofs is satisfactory.

*Disagree*      *Agree*      *Strongly Agree*

### Goal 3 Outcome 1

**Assessment Problem:** Students will be given two assessment questions, one multiple-choice and the other free-response, to assess their abilities to read computer programs.

1. What is the output of the following code?

```
x = 1.5
if x < 2.1 or x > 3.0:
    print(x*2)
elif x == 1.5:
    print(x*3)
else:
    print(x*4)
```

2. Consider the following output below. Which of the following code will produce this output?

```
11111
2222
333
44
5
```

- a) 

```
for i in range(1, 6):
    for k in range(1, i+1):
        print(i, end='')
    print()
```
- b) 

```
for i in range(1, 6):
    for k in range(1, i+1):
        print(k, end='')
    print()
```
- c) 

```
for i in range(1, 6):
    for k in range(1, 6):
        print(i, end='')
    print()
```
- d) 

```
for i in range(1, 6):
    for k in range(1, 7-i):
        print(i, end='')
    print()
```
- e) 

```
for i in range(1, 6):
    for k in range(1, 6):
        print(k, end='')
    print()
```

**Programming Performance rubric for Outcome 1:**

- 1 – Does not meet faculty expectations
  - Neither of the two assessment questions is answered correctly.
- 2 – Meets faculty expectations
  - Exactly one of the two assessment questions is answered correctly.
- 3 – Exceeds faculty expectations
  - Both of the two assessment questions are answered correctly.

**Goal 3 Outcome 2**

**Assessment Problem:** Students will be asked to write a computer problem to solve a mathematical problem.

Define explicitly a function named **func** that returns the value of  $7x - 3x^2$  for input argument  $x$ . Then write a **for** loop to find the maximum value of **func** and its corresponding  $x$ -value when **func** is evaluated at  $x = 0.1, 0.2, 0.3, \dots, 10.0$ . Finally, display the maximum value followed by its corresponding  $x$ -value.

**Programming Performance rubric for Outcome 2:**

- 1 – Does not meet faculty expectations
  - Response contains major logical and/or syntax errors.
- 2 – Meets faculty expectations
  - Response contains minor logical and/or syntax errors.
- 3 – Exceeds faculty expectations
  - Response contains no logical errors and at most two minor syntax errors.

**Goal 3 Outcome 3**

How would you respond to the following statement: My ability to use appropriate technology to solve mathematical problems is satisfactory.

*Disagree      Agree      Strongly Agree*

**Goal 4 Outcome 1**

How would you respond to the following statement: I have an appreciation for the beauty of mathematics as a singular discipline.

*Disagree*      *Agree*      *Strongly Agree*

**Goal 4 Outcome 2**

How would you respond to the following statement: I have an appreciation for mathematics in its application in real world applications.

*Disagree*      *Agree*      *Strongly Agree*

## **Goal 5 Outcome 1**

### **Assessment Problem:**

Students will submit a report and/or display a poster presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

1. The student clearly states the mathematical topic to be examined.
2. The report/poster shows an understanding of the proof or methods used.
3. The student is able to extend results, hypothesize ramifications of the result, and/or apply the result in the appropriate setting.

### **Communication Performance rubric for Outcome 1:**

Failure in any of the above requirement will result in a 1. Satisfactory completion of all three requirements will result in a 2.

Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional poster presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, and acceptable display formatting.

## **Goal 5 Outcome 2**

### **Assessment Problem:**

Students will make a presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

1. The student has a clear understanding of what is set to be proved.
2. The student shows an understanding of the proof by successfully summarizing methods.
3. The student is able hypothesize ramifications of the theorem and/or apply the theorem in the appropriate setting.

### **Communication Performance rubric for Outcome 2:**

Failure in any of the above requirements will result in a 1. Satisfactory completion of all three requirements will result in a 2.

Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, acceptable degrees of audience interaction, and possibly extend results.

### **Goal 5 Outcome 3**

#### **Assessment Problem:**

Students will write a lesson plan describing in detail the objectives, methods, and outcomes of a class lesson.

#### **Communication Performance rubric for Outcome 3:**

1 – Does not meet faculty expectations

\*Lesson plan did not contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles.

\* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles in a limited fashion. Accommodations contained less than sufficient details/steps.

2 – Meets faculty expectations

\*Lesson plan contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles. Accommodations contained sufficient details/steps.

3 –Exceeds faculty expectations

\* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles. Accommodations contained more than sufficient details/steps.

### **Goal 5 Outcome 4**

How would you respond to the following statement: I am able to effectively communicate mathematics in written form and oral presentations.

*Disagree*

*Agree*

*Strongly Agree*