Name of Program/Department:	Mathematics Program
Year:	2019-20
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# **Program Mission Statement**

A primary purpose of the Department of Mathematics at Francis Marion University is to offer all University students a varied and well-balanced curriculum of undergraduate education in mathematics. In the liberalarts tradition, the courses in the curriculum teach students to think logically, to analyze problems and solve them appropriately, and to communicate their ideas clearly.

The department also provides a broad range of entry-level courses in order to meet the needs of students with widely varying mathematical backgrounds and to provide them with skills appropriate for their selected majors. The mathematics courses that satisfy the General Education requirement in mathematics are designed to help students achieve *Goal 5: The ability to use fundamental mathematical skills and principles in various applications*.

Equally important, the curriculum provided by the Department leads to baccalaureate degrees in two distinct but overlapping areas: mathematical sciences and teacher licensure in mathematics. These courses prepare students for careers in education, business, industry, and government. They also prepare those students of sufficient interest and ability for further study of mathematics at the graduate level.

# **Program Learning Outcomes**

- 1. Students should be able to analyze problems involving various applications and solve them using appropriate mathematical skills, principles, and technology.
- 2. Students should be able to present oral and written solutions in a structured format that can be understood by a general audience.
- 3. Students should recognize and appreciate the applicability, beauty, and power of mathematics.
- 4. Students should be confident in their abilities to use mathematics to solve various problems.

# **Executive Summary**

The Department of Mathematics uses several direct and indirect assessments. The direct assessments of Student Learning Outcomes (SLO) 1.0 (Outcomes 1-4), SLO 2.0 (Outcomes 1-2), SLO 3.0 (Outcomes 1-2), and SLO 5.0 (Outcomes 1-2) are evaluated using a calculus performance rubric, an elementary proof performance rubric, a technology usage performance rubric, and a communication performance rubric from student work samples. Values are the percentages of students who met or exceeded faculty expectations based on the rubrics. The indirect assessments of SLO 1.5<sup>†</sup>, SLO 2.3, SLO 3.3, SLO 4.1-4.2, and SLO 5.4 are tabulated from student attitude surveys. Values are percentages of students who state that they are confident in their skills and abilities or have an appreciation for the beauty of mathematics as a singular discipline and its applications.

Academic year 2019-20 assessments show targets were achieved in 15 of 17 outcomes. It appears that revisions to assessment problems in SLO 1.1 and 1.3 have been successful in the attempt to more accurately assess students' performance. Instructors felt that students were avoiding previous assessment problems because they were tedious. This is the first year that SLO 1.1 achieved the target. Assessment scores in SLO 2.1-2.2 rebounded in 2019-20 from low scores last year. The decrease in 2018-19 scores was disproportionately evident in Math 230. Students are meeting targets in the demonstration of applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching). Since assessment of the demonstration of applications of various strategies and tools has exceeded the target of 70 the last two years, the target for this outcome (SLO 5.3) will be increased to 80 in 2020-21.

Targets were not achieved in 2 of 17 outcomes. Direct assessment SLO 2.1 did not achieve the target. We will continue to watch this assessment to see if the increase will result in achieved targets in future years. Instructors of Math 230/311 wish to consider reducing the number of assessment problems in SLO 2.1-2.2. They feel that the current number of assessment problems is excessive and that requiring fewer problems can still accurately assess students' performance. Direct assessment SLO 3.2 had achieved the target the previous three years but not in 2019-20. A change in the programming language from FORTRAN to Python required revised assessment problems in SLO 3.1-3.2. We will watch this assessment due to the change in programming languages.

Lastly, assessment results for the fall and spring semesters are displayed in two columns in Table 1.0. Distancing restrictions due to COVID-19 required the last five weeks of the semester to be completed online including most of the assessments. This change might have allowed students to use resources not available to them in the classroom setting. Even without spring semester scores, the fall averages exceeded the previous year's average in SLO 1.1-1.3.

# **Student Learning Outcomes**

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Students will demonstrate competence to calculate derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499). Outcome 2: Students will demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499). Outcome 3: Students will demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499). Outcome 4: Students will demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499).

Outcome 5: Students will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence (Math 499).

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Students will be able to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311). Outcome 2: Students will be able to write elementary proofs (Math 230/311). Outcome 3: Students will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs (Math 499).

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Students will be able to read computer programs that model various mathematical applications (Math 213).

Outcome 2: Students will be able to write computer programs that model various mathematical applications (Math 213).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems (Math 499).

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Students will respond to a statement concerning their appreciation for the beauty of mathematics as a singular discipline (Math 499).

Outcome 2: Students will respond to a statement concerning their understanding of the importance of mathematics in real world applications (Math 499).

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Students will communicate mathematics in a written presentation (Math 499). Outcome 2: Students will communicate mathematics in an oral presentation (Math 499). Outcome 3: Secondary education students will demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching). Outcome 4: Students will respond to a statement concerning their confidence in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499).

# Assessment Methods

The direct assessments (SLO 1.1-1.4, SLO 2.1-2.2, SLO 3.1.3.2, and SLO 5.1-5.2) are evaluations of student work samples normally part of the end of course exam using performance rubrics for calculus, elementary proof, technology usage, and communication. The indirect assessments (SLO 1.5, SLO 2.3, SLO 3.3, SLO 4.1-4.2, and SLO 5.4 are tabulated from a senior survey administered in Math 499 at the end of the semester.

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

For outcomes 1-4, instructors of Calculus sequence courses (Math 201, 202, 203, 306) and Mathematics Capstone Course (Math 499) will collect samples of student solutions to problems or other work that call for students to demonstrate proficiency of basic computational techniques in the calculus sequence. Student solutions will be evaluated based on a calculus performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target is for 70% of students to meet or exceed faculty expectations. For outcome 5, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

For outcomes 1-2, instructors of Discrete Mathematics I (Math 230) and Transition to Higher Mathematics (Math 311) will collect samples of student solutions or relevant problems of other work to demonstrate the ability to understand and construct elementary proofs. Student solutions will be evaluated based on a proof performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target is for 70% of students to meet or exceed faculty expectations. For outcome 3, students will complete a senior survey in the

Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

For outcomes 1-2, instructors of Scientific Programming in Python (Math 213) will collect samples of student solutions to relevant problems or other work to demonstrate the ability to use appropriate technology to solve mathematical problems. Student solutions will be evaluated based on a programming performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target is for 70% of students to meet or exceed faculty expectations. For outcome 3, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree. The target is for 95% of students to agree or strongly agree.

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Students will complete senior surveys in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree to statements concerning their appreciation for the beauty of mathematics and their understanding of the importance of mathematics. The target is for 95% of students to agree or strongly agree.

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

For outcomes 1-3, instructors of the Mathematics Capstone Course (Math 499) and supervisors of student teachers will collect samples of student work and will attend presentations that call for students to effectively communicate mathematics. Student work and presentations will be evaluated based on a communication performance rubric (1 = does not meet faculty expectations; 2 = meets faculty expectations; 3 = exceeds faculty expectations). The target for outcomes 1-2 is for 80% of students to meet or exceed faculty expectations. The target for outcome 3 is for 70% of students to meet or exceed faculty expectations. For outcome 4, students will complete a senior survey in the Mathematics Capstone Course (Math 499) with responses of disagree, agree, and strongly agree.

# Assessment Results

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Seventy-two of ninety (80.0%) students did demonstrate competence to calculate derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499). This was a substantial increase from previous years. This target was achieved.

Outcome 2: Thirty-two of forty-four (72.7%) students did demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499). This is a substantial increase from previous year. This target was achieved.

Outcome 3: Thirty of thirty-nine (76.9%) students did demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499). This was a substantial increase from previous years. This target was achieved.

Outcome 4: Thirty-four of thirty-six (94.4%) students did demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499). This is a slight decrease from last year. The target was achieved.

Outcome 5: Five of five (100.0%) students did respond that they were confident in their computational techniques in the calculus course sequence (Math 499). This target was achieved.

SLO 1.1-1.4 achieved the target of 70%. SLO 1.5 achieved the target of 95%. SLO 1.0's overall target was achieved.

SLO 2.0: Students in Math 230 and 311 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Fifteen of twenty-three (65.2%) students did show ability to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311). This target was not achieved.

Outcome 2: Nineteen of twenty-three (82.6%) students did show ability to write elementary proofs (Math 230/311). This target was achieved.

Outcome 3: Five of five (100.0%) students did respond that they were confident in their ability to understand and construct elementary proofs (Math 499). This target was achieved.

SLO 2.1 was below the target of 70%. SLO 2.2 achieved the target of 70%. SLO 2.3 achieved the target of 95%. SLO 2.0's overall target was not achieved.

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Fifteen of seventeen (88.2%) students did show ability to read computer programs that model various mathematical applications (Math 213). This target was achieved.

Outcome 2: Eleven of seventeen (64.7%) students did show ability to write computer programs that model various mathematical applications (Math 213). This target was not achieved.

Outcome 3: Five of five (100.0%) students did respond that they were confident in their ability to use appropriate technology to solve mathematical problems (Math 499). This target was achieved.

SLO 3.1 achieved the target of 70%. SLO 3.2 did not achieve the target of 70%. SLO 3.3 did achieve the target of 95%. SLO 3.0's overall target was not achieved.

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Five of five (100.0%) students did respond that they had an appreciation for the beauty of mathematics as a singular discipline (Math 499). This target was achieved.

Outcome 2: Five of five (100.0%) students did respond that they had an understanding of the importance of mathematics in real world applications (Math 499). This target was achieved.

SLO 4.0's overall target was achieved.

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Five of five (100.0%) students did communicate mathematics effectively in a written presentation (Math 499). This target was achieved.

Outcome 2: Five of five (100.0%) students did communicate mathematics effectively in an oral presentation (Math 499). This target was achieved.

Outcome 3: One of one (100.0%) student did demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching). This target was achieved.

Outcome 4: Five of five (100.0%) students did respond that they were confident in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499). This target was achieved.

SLO 5.0's overall target was achieved.

Assessment	2016-17	2017-18	2018-19	Fall 2019	Spring 2020	2019-20
SLO 1.0 Outcome 1	34.3	46.1	47.9	72.0	90.0	80.0
Outcome 2	58.7	56.1	50.7	66.7	76.9	72.7
Outcome 3	48.1	51.2	67.4	82.4	72.7	76.9
Outcome 4	68.8	88.2	94.7	92.3	95.7	94.4
Outcome 5	100.0	100.0	100.0		100.0	100.0
SLO 2.0 Outcome 1	86.4	76.5	57.1	100.0	60.0	65.2
Outcome 2	63.6	82.4	46.4	100.0	80.0	82.6
Outcome 3	100.0	100.0	100.0		100.0	100.0
SLO 3.0 Outcome 1	73.9	76.5	87.5	80.0	100.0	88.2
Outcome 2	73.9	70.6	87.5	40.0	100.0	64.7
Outcome 3	100.0	100.00	88.9		100.0	100.0
SLO 4.0 Outcome 1	100.0	100.0	100.0		100.0	100.0
Outcome 2	100.0	100.0	100.0		100.0	100.0
SLO 5.0 Outcome 1	81.8	85.7	88.9		100.0	100.0
Outcome 2	81.1	71.4	88.9		100.0	100.0
Outcome 3	*1	*1	100.0		100.0	100.0
Outcome 4	100.0	100.0	100.0		100.0	100.0

# Table 1.0: Assessment Results

\*1 No students participated in student teaching during the academic year.

# Action Items

SLO 1.0: Students in Math 201, 202, 203, 306, and 499 will be proficient in the elementary computational techniques in the calculus course sequence. Students in Math 499 will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence.

Outcome 1: Students will demonstrate competence to calculate derivatives and use them in one or more applications, such as optimization or related rates problems (Math 201/499). Outcome 2: Students will demonstrate competence to calculate integrals and use them in various applications, such as area, volume, or average value of a function over an interval (Math 202/499). Outcome 3: Students will demonstrate competence to calculate convergence of series and use them in various applications, such as polynomials to approximate functions (Math 203/499). Outcome 4: Students will demonstrate competence to calculate gradients and partial derivatives and use them in various applications (Math 306/499).

Outcome 5: Students will respond to a statement concerning their confidence in their computational techniques in the calculus course sequence (Math 499).

The assessment problems of Outcomes 1 and 3 were revised before the 2019-20 academic year because instructors felt that the problems did not accurately measure student performance. This revision seems initially to be successful based on 2019-20 data.

SLO 2.0: Students in Math 230 will develop the ability to understand and construct elementary proofs. Students in Math 499 will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs.

Outcome 1: Students will be able to read and understand elementary proofs and be able to determine what constitutes a mathematical proof (Math 230/311). Outcome 2: Students will be able to write elementary proofs (Math 230/311).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to understand and construct elementary proofs (Math 499).

Assessment scores in Outcomes 1-2 rebounded in 2019-20 from low scores last year. The decrease in 2018-19 scores was disproportionately evident in Math 230. Instructors of Math 230/311 wish to decrease the number of assessment problems in Outcomes 1-2. They feel the assessment problems require most of the allotted final exam completion time and leave little time for other problems. Additionally, fewer problems might lead to more consistence in scores.

SLO 3.0: Students in Math 213 will be able to use appropriate technology to solve mathematical problems. Students in Math 499 will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems.

Outcome 1: Students will be able to read computer programs that model various mathematical applications (Math 213).

Outcome 2: Students will be able to write computer programs that model various mathematical applications (Math 213).

Outcome 3: Students will respond to a statement concerning their confidence in their ability to use appropriate technology to solve mathematical problems (Math 499).

The programming language used in the computer programing course was changed from FORTRAN to Python. This required a course change from Math/CS 212 Introduction to FORTRAN to Math 213 Scientific Programming in Python. Assessment problems in Outcomes 1-2 were revised to accommodate this change in programming languages. We will continue to watch this assessment measure to be sure the revised assessment problems accurately assess students' understanding.

SLO 4.0: Students in Math 499 will appreciate the beauty of mathematics as a singular discipline and its applications.

Outcome 1: Students will respond to a statement concerning their appreciation for the beauty of mathematics as a singular discipline (Math 499).

Outcome 2: Students will respond to a statement concerning their understanding of the importance of mathematics in real world applications (Math 499).

These targets were met and no action is planned.

SLO 5.0: Students in Math 499 and Student Teaching will be able to effectively communicate mathematics in written form and oral presentations.

Outcome 1: Students will communicate mathematics in a written presentation (Math 499). Outcome 2: Students will communicate mathematics in an oral presentation (Math 499). Outcome 3: Secondary education students will demonstrate applications of various strategies and tools in the teaching of mathematical concepts (Student Teaching). Outcome 4: Students will respond to a statement concerning their confidence in their ability to develop and effectively communicate mathematics in written form and oral presentations (Math 499).

These targets were met. Due to achieved targets in Outcome 3, the target will be increased to 80.

# Appendices

#### **Goal 1 Outcome 1**

#### **Assessment Problem 1:**

Evaluate  $\lim_{x\to 2} \frac{x^2 - 3x + 2}{x^3 + 5x^2 - 14x}$  (any indeterminate limit that requires algebraic manipulation is suitable)

Score	Criteria
1	Incorrect approach. Evaluated limit incorrectly as written.
2	Correct approach (either algebraic manipulation or L 'Hospital's rule) but calculation
	mistake in algebra or arithmetic.
3	Correct limit using appropriate calculus.

#### **Assessment Problems 2 and 3:**

Two differentiation problems that require use of at least two of the following differentiation rules: chain rule, product rule and quotient rule. Examples of such problems are as follows:

 $\frac{x \cdot e^x}{\cos x}, \ \frac{x \cdot \cot x}{7^x}, \ \ \frac{\ln x^2}{x}, \ \ (2x^3 + 4x^{-1} + 3)^{10} \cdot \log_3 x, \ \ x^2 \cdot \arctan 5x, \ \ \sin(\cos x^4)$ 

#### Likert scale for Problems 2 and 3:

Score	Criteria
1	Incorrect approach and/or calculus errors.
2	Properly identified rules required for the solution; algebra, arithmetic and/or notation
	errors.
3	Correct answer with proper notation.

#### **Assessment Problem 4:**

Use the Extreme Value Theorem to find the absolute maximum and absolute minimum of a cubic polynomial on a closed interval.

#### Likert scale for Problem 4:

Score	Criteria
1	Incorrect approach, incorrect derivative, critical values incorrect or incorrect
	mathematics used in finding them and/or no calculus used.
2	Correct critical values found. Found local maximum/minimums using critical points but
	does not check endpoints.
3	Properly finds absolute maximum and minimum using EVT and correctly identifies
	critical numbers.

# Total sum of 4-7 produces an overall score of 1, 8-10 produces an overall score of 2, and 11-12 produces an overall score of 3.

Find the area of the shaded region.

(0,4)

1)

Goal 1 Outcome 2

**Assessment Problems:** 



(3, -5)

1 - Does not meet faculty expectations

\*Integral set up incorrectly.

\*Integral set up correctly but evaluated incorrectly at a conceptual level such as the incorrect use of the Evaluation Theorem or the absolute value of the integral is not taken.

2 - Meets faculty expectations

\*Integrals are set up properly.

\*Integrals are evaluated properly with possible minor calculation errors.

\*Absolute value is taken to reflect the area of the entire shaded region.

#### 3 - Exceeds faculty expectations

\*Correct solution with proper work shown and no calculation errors.

#### Use the substitution formula to evaluate the integral.

2) 
$$\int_{-1}^{0} \frac{2t}{(3+t^2)^3} dt$$

#### Evaluate the integral.

$$3) \int 5x \sin x \, dx$$

# Calculus Performance rubric for Outcome 2 for Problems 2 and 3:

Score	Criteria
1	Incorrect approach
	Major calculation errors
2	• Properly identified rules required for the solution
	• Minor calculation errors including algebra and notation
3	Correct answer with proper notation

# Total sum of 3-5 produces an overall score of 1, 6-7 produces an overall score of 2, and 8-9 produces an overall score of 3. Goal 1 Outcome 3

#### **Assessment Problems:**

1. Describe in your own words what it means for a series  $\sum a_n$  to converge. Does the convergence of the sequence  $\{a_n\}$  imply the convergence of the series  $\sum a_n$ ?

Score	Criteria
1	Answered both questions incorrectly.
2	Answered one question correctly.
3	Answered both questions correctly.

2. (Choose one) Determine whether the series converges or diverges. You can use whatever test that you prefer. However, you must show how you use the test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$$

$\sum_{n=1}^{\infty}$	$2^n$
$\sum_{n=0}$	$\overline{3^n+1}$

Score	Criteria
1	Incorrect approach or no work shown.
2	Correct approach but some algebraic/arithmetic errors or insufficient work shown.
3	Correct approach with the required work shown.

3. Determine the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

Score	Criteria
1	Incorrect approach or no work shown.
2	Correct approach but some algebraic/arithmetic errors or insufficient work shown.
3	Correct approach with the required work shown.

4. Let  $f(x) = e^{3x}$ . Find the Taylor series for the function f centered at 0. List the first six terms of the series and then write the series using  $\Sigma$  notation.

Score	Criteria
1	Answered both parts incorrectly.
2	Answered one part correctly.
3	Answered both parts correctly.

Total sum of 4-7 produces an overall of 1, 8-10 produces an overall score of 2, and 11-12 produces an overall score of 3.

#### **Goal 1 Outcome 4**

#### **Assessment Problem:**

Let  $f \in \mathbb{R}^2 \to \mathbb{R}^1$  with  $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y + 2$ . Compute the following functions, numbers, vector, and equation by hand.

- (a)  $\frac{\partial}{\partial x} f(x, y) =$
- (b)  $eval_{(x,y)=(1,2)} \frac{\partial}{\partial x} f(x,y) =$
- (c)  $\frac{\partial}{\partial y} f(x, y) =$
- (d)  $eval_{(x,y)=(1,2)} \frac{\partial}{\partial y} f(x,y) =$
- (e)  $\nabla f(1,2) =$
- (f, g) Find the equation of the plane  $\Pi$  tangent to the surface  $\{z = f(x, y)\}$  through the point (1, 2, 21).

#### **Calculus Performance rubric for Outcome 4:**

1 (Not meet expectation) Student cannot find partial derivatives and evaluate them at the point specified.

2 (Meets expectation) Student can find partial derivatives and evaluate them at the point specified.

3 (Exceeds expectation) Student goes on to find the gradient and equation of the tangent plane.

#### **Goal 1 Outcome 5**

How would you respond to the following statement: I am proficient in elementary computational techniques in the calculus.

#### **Goal 2 Outcome 1**

#### **Assessment Problem:**

For each proposition stated, if the proof given is in fact a valid proof, identify the method of proof used (Direct, Indirect Contrapositive, Contradiction, or Induction). If there is an error in the proposed proof, find and describe the problem. An error could be any of the following: the use of a fallacy argument, an algebraic error, the misuse of a definition, an incomplete proof, and so on.

**Proposition 1.** If x and y are integers which are either both even or both odd, then x - y is even.

*Proof.* Let x and y be integers. We consider two cases, according to whether x and y are both even or both odd.

Case 1: x and y are both even. Let x = 6 and y = 2, which are are both even. Then x - y = 4, which is even.

Case 2: x and y are both odd. Let x = 7 and y = 1, which are both odd. Then x - y = 6, which is even.

**Proposition 2.** If m is an even integer and n is an odd integer, then 3m + 5n is odd.

*Proof.* Let m be an even integer and let n be an odd integer. Then m = 2k and n = 2k + 1, for some  $k \in \mathbb{Z}$ . Therefore, we have

$$3m + 5n = 3(2k) + 5(2k + 1)$$
  
=  $6k + 10k + 5$   
=  $16k + 5$   
=  $2(8k + 2) + 1$ .

Since 8k + 2 is an integer, it follows that 3m + 5n is odd.

Proposition 3. For all  $x \in \mathbb{R}$ , |x| = 2 if and only if  $x^2 - 4 = 0$ . *Proof.* Case 1: Consider any  $x \in \mathbb{R}$  for which |x| = 2. Then  $|x|^2 = 2^2 = 4$ . As  $|x|^2 = x^2$  for any  $x \in \mathbb{R}$ , it follows that  $x^2 - 4 = 4 - 4 = 0$ .

Case 2: Consider any  $x \in \mathbb{R}$  for which  $x^2 - 4 \neq 0$ . Then  $x^2 \neq 4$ , and so  $\sqrt{x^2} \neq \sqrt{4}$ . This implies  $|x| \neq 2$ , and so this case follows by contrapositive.

**Proposition 4.** For all  $x \in \mathbb{Z}$ , if  $x^2 + 2x$  is even then x is even.

*Proof.* Consider any  $x \in \mathbb{Z}$  such that x is odd. Then x = 2k + 1 for some  $k \in \mathbb{Z}$ . It follows that

$$x^{2} + 2x = (2k + 1)^{2} + 2(2k + 1) = 4k^{2} + 4k + 1 + 4k + 2$$
$$= (4k^{2} + 8k + 2) + 1$$
$$= 2(2k^{2} + 4k + 1) + 1.$$

Therefore,  $x^2 + 2x$  is an odd integer, and the proposition follows.  $\Box$ 

Proposition 5. For all  $n \in \mathbb{N}$ ,  $\sum_{j=1}^{n} (2j-1) = n^2$ .

*Proof.* If n = 1, then

$$\sum_{j=1}^{n} (2j-1) = 2(1) - 1 = 1 = 1^2 = n^2.$$

Thus, the statement is true when n = 1. Now suppose that the statement is true when n = k for any  $k \ge 1$ , and thus

$$\sum_{j=1}^{k} (2j-1) = k^2 . (*)$$

Then for n = k + 1, we have

$$\sum_{j=1}^{n} (2j-1) = \sum_{j=1}^{k+1} (2j-1) = \sum_{j=1}^{k} (2j-1) + \sum_{j=k+1}^{k+1} (2j-1)$$
$$= \sum_{j=1}^{k} (2j-1) + [(2(k+1)-1]]$$
$$= k^2 + [2(k+1)-1] \quad \text{by (*) above}$$
$$= k^2 + 2k + 1 = (k+1)^2 = n^2.$$

Thus, if the statement is true when n = k for any  $k \ge 1$ , the statement has also been shown to be true for n = k+1. Therefore, the statement must be true for all  $n \in \mathbb{N}$ .

# **Proof Performance rubric for Outcome 1:**

Outcome 1

(1) 0-2 correctly identified proposition(s) does not meet faculty expectations.

(2) 3-4 correctly identified propositions meets faculty expectations.

(3) 5 correctly identified propositions exceeds faculty expectations.

# Goal 2 Outcome 2

### **Assessment Problem:**

Provide mathematically correct, properly written proofs for the given statements, using the indicated proof technique if so directed.

Assessment Problem: Provide mathematically correct, properly written proofs for the given statements.

- (1) Let X be a non-empty set, and let A, B, and C be subsets of X. Prove that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .
- (2) For all  $x, y \in \mathbb{Z}$ , prove that x + 3y is even if and only if x and y are either both even or both odd.
- (3) Prove that, for every positive integer n,

$$\sum_{j=1}^{n} 4j + 6 = 2n(n+4).$$

(4) Prove that, for any  $x \in \mathbb{R}$ , if  $x^2(x+1)$  is irrational then x is irrational.

# **Proof Performance rubric for Outcome 2:**

Problems 1, 3, and 4 count for 1 proof and Problem 2 counts for 2 proofs:

(1) 0-2 proven correctly: does not meet faculty expectations.

(2) 3-4 proven correctly: meets faculty expectations.

(3) 5 proven correctly: exceeds faculty expectations.

# Goal 2 Outcome 3

How would you respond to the following statement: My ability to understand and construct elementary proofs is satisfactory.

#### **Goal 3 Outcome 1**

**Assessment Problem:** Students will be given two assessment questions, one multiple-choice and the other free-response, to assess their abilities to read computer programs.

1. What is the output of the following code?

```
x = 1.5
if x < 2.1 or x > 3.0:
    print(x*2)
elif x == 1.5:
    print(x*3)
else:
    print(x*4)
```

2. Consider the following output below. Which of the following code will produce this output?

```
11111
2222
333
44
5
```

```
a) for i in range(1, 6):
       for k in range(1, i+1):
           print(i, end='')
       print()
b) for i in range(1, 6):
       for k in range(1, i+1):
           print(k, end='')
       print()
c) for i in range(1, 6):
       for k in range(1, 6):
           print(i, end='')
       print()
d) for i in range(1, 6):
       for k in range(1, 7-i):
           print(i, end='')
       print()
e) for i in range(1, 6):
       for k in range(1, 6):
           print(k, end='')
       print()
```

#### **Programming Performance rubric for Outcome 1:**

- 1 Does not meet faculty expectations
  - Neither of the two assessment questions is answered correctly.
- 2 Meets faculty expectations
  - Exactly one of the two assessment questions is answered correctly.
- 3 Exceeds faculty expectations
  - Both of the two assessment questions are answered correctly.

#### Goal 3 Outcome 2

Assessment Problem: Students will be asked to write a computer problem to solve a mathematical problem.

Define explicitly a function named **func** that returns the value of  $7x - 3x^2$  for input argument x. Then write a **for** loop to find the maximum value of **func** and its corresponding x-value when **func** is evaluated at x = 0.1, 0.2, 0.3, ..., 10.0. Finally, display the maximum value followed by its corresponding x-value.

#### **Programming Performance rubric for Outcome 2:**

- 1 Does not meet faculty expectations
  - Response contains major logical and/or syntax errors.
- 2 Meets faculty expectations
  - Response contains minor logical and/or syntax errors.
- 3 Exceeds faculty expectations
  - Response contains no logical errors and at most two minor syntax errors.

#### Goal 3 Outcome 3

How would you respond to the following statement: My ability to use appropriate technology to solve mathematical problems is satisfactory.

#### **Goal 4 Outcome 1**

How would you respond to the following statement: I have an appreciation for the beauty of mathematics as a singular discipline.

Disagree Agree Strongly Agree

#### Goal 4 Outcome 2

How would you respond to the following statement: I have an appreciation for mathematics in its application in real world applications.

#### **Goal 5 Outcome 1**

#### **Assessment Problem:**

Students will submit a report and/or display a poster presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

- 1. The student clearly states the mathematical topic to be examined.
- 2. The report/poster shows an understanding of the proof or methods used.
- 3. The student is able to extend results, hypothesize ramifications of the result, and/or apply the result in the appropriate setting.

#### **Communication Performance rubric for Outcome 1:**

Failure in any of the above requirement will result in a 1. Satisfactory completion of all three requirements will result in a 2.

Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional poster presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, and acceptable display formatting.

#### Goal 5 Outcome 2

#### **Assessment Problem:**

Students will make a presentation at an undergraduate mathematics conference (or other suitable venue) of a mathematical research project that satisfies the following requirements:

- 1. The student has a clear understanding of what is set to be proved.
- 2. The student shows an understanding of the proof by successfully summarizing methods.
- 3. The student is able hypothesize ramifications of the theorem and/or apply the theorem in the appropriate setting.

#### **Communication Performance rubric for Outcome 2:**

Failure in any of the above requirements will result in a 1. Satisfactory completion of all three requirements will result in a 2.

Furthermore, a student whose presentation is deemed exceptional by a majority of the department faculty present will receive a 3. Exceptional presentations include clear understanding of the material such that the student is not reliant on materials, ability to answer pertinent questions, acceptable degrees of audience interaction, and possibly extend results.

#### **Goal 5 Outcome 3**

#### **Assessment Problem:**

Students will write a lesson plan describing in detail the objectives, methods, and outcomes of a class lesson.

#### **Communication Performance rubric for Outcome 3:**

1 – Does not meet faculty expectations

\*Lesson plan did not contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles.

- \* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles in a limited fashion. Accommodations contained less than sufficient details/steps.
- 2 Meets faculty expectations

\*Lesson plan contain instructional strategies that accommodate for visual, auditory and kinesthetic learning styles. Accommodations contained sufficient details/steps.

3 –Exceeds faculty expectations

\* Lesson plan contained instructional strategies that accommodated for visual, auditory and kinesthetic learning styles. Accommodations contained more than sufficient details/steps.

#### Goal 5 Outcome 4

How would you respond to the following statement: I am able to effectively communicate mathematics in written form and oral presentations.