THE · 43RD · PEE – DEE · REGIONAL · HIGH – SCHOOL

MATHEMATICS · TOURNAMENT

Written Competition

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MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER

TUESDAY · 2019 · DECEMBER · 03

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name in the space below
as you wish it to appear if you are given an award.

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored. If a problem specifies a certain form for an answer, then your answer *must* conform in order to receive credit.

Because the judges must score over 300 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	**Acceptable**
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$

Name of student

Name of high school

Awards

Page	Problems	Number Correct
2	1 2 3 4	
3	5 6 7	
4	8 9 10 11	
5	12 13 14	
6	15 16 17 18	
Total		

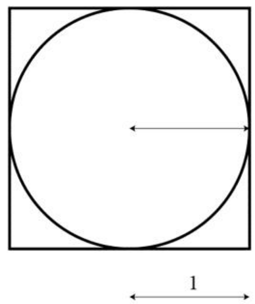
In the following problems, your answers must be exact. Do not use 3.14 as an approximation for π. Do not attempt to rationalize a denominator with π in it; you can't.

1. A circle of radius 1 units is inscribed in a square. What is the area of the square?

2. What is the area of the circle?

3. How many times larger is the area of the square than the area of the circle?

4. If a larger circle were circumscribed about the square of the previous problems, how many times larger would its area be than the area of the original circle?



— In order to receive credit, answers must appear in these boxes and be thoroughly simplified. —

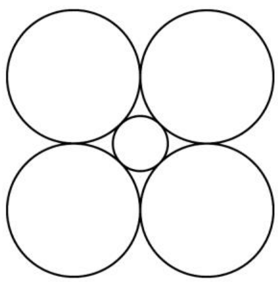
Answer to Problem 1:	Answer to Problem 2:	Answer to Problem 3:	Answer to Problem4:
square units	square units		

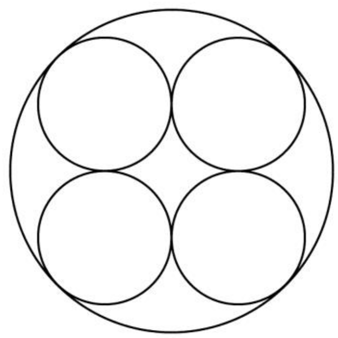
All final answers to problems on this page must be thoroughly simplified.

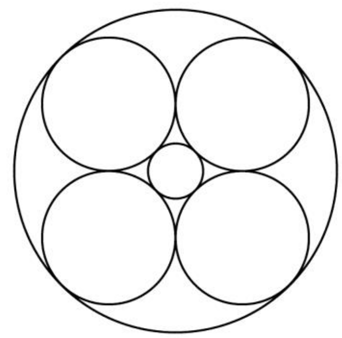
5. Four circles of equal size are tangent to one another in a square arrangement. A smaller circle, nested inside, is tangent to the four original circles. How many times more area does each of the original circles have than the area of the smaller circle?

6. Four circles of equal size are arranged as before. Now, a big circle circumscribes the arrangment of original circles, as shown. How many times larger is the area of the big circle than the area of any one of the original circles?

7. How many times larger is the area of the big circle in Problem 6 than the area of the small circle in Problem 5?







— In order to receive credit, answers must appear in these boxes and thoroughly simplified. —

Answer to Problem 5:	**Answer to Problem 6:**	**Answer to Problem7:**

Recall that if a number is positive, it cannot be zero. All final answers to problems on this page must be thoroughly simplified.

8. Once upon a time, there were two positive numbers, though some people thought they were only one number, because they were the same number in that they were equal to one another. The product of these two numbers was equal to the sum of the two numbers. What was their product, which is the same as their sum?

9. On the next day, there were also two positive numbers, and this time no one thought they were the same number, because one number was one larger than the other number. The product of these two numbers was also equal to their sum. What was the sum of these two numbers, which is the same as their product?

10. A week passed, and two more numbers came onto the scene. Only a fool would think they were the same number, because one number was positive and the other number was negative. Furthermore, one of the numbers was one larger than the other number. Even so, the product of these numbers was equal to the sum of these numbers. What was the product of these numbers, which is the same as the answer to this problem?

11. Three numbers walked into a bar, and nobody liked them because all three of them were negative. The largest number was one more than than the middle number, which in turn was one more than the smallest number. You might think that their product equaled their sum, but you would be wrong, because the product of the three numbers was equal to their *average.* What was the product of these numbers, which was their average, which is the answer to this problem?

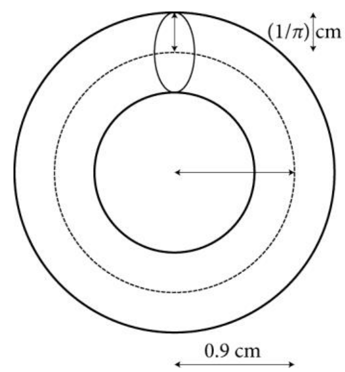
— In order to receive credit, answers must appear in these boxes and be thoroughly simplified. —

Answer to Problem 8:	**Answer to Problem 9:**	**Answer to Problem 10:**	**Answer to Problem 11:**

You probably do not know the formula for the volume of a torus, but if you are good at mathematics, you can think of a formula that is probably right, and this formula has been shown to be correct. If you cannot solve #12, you may still be able to solve #13 and thus #14.

12. Stud-Man went to a jewelry store to buy an engagement ring for Daisy-Belle. The owner came into the store and announced that everything in the store was being sold at a 30% discount that day. After Stud-Man made his selection, the clerk (not the owner) deducted one-third from the original price of the ring. Stud-Man ended up paying $9.00 less than under the owner's "30%-off" policy. How much did Stud-Man pay for the ring? *You must answer in dollars and cents, rounded, if needed, to the nearest cent.*

13. The ring Stud-Man purchased was in the shape of a torus. Stud-Man estimated the major radius to be 0.9 centimeter, and the minor radius to be $1/\pi$ centimeters. Upon assuming that these estimates were accurate, what was the physical volume of the ring? *You must answer in the units specified in the answer-box.*



14. The ring Stud-Man purchased was made of solid silver and was the size and shape as in the last problem. Silver has a density of 10.5 grams / cubic centimeter and a price of $500.00 per kilogram. What was the worth of the silver in the ring Stud-Man purchased? *You must answer in dollars and cents, rounded, if needed, to the nearest cent.*

— In order to receive credit, answers must appear in these boxes and ithoroughly simplified. —

Answer to Problem 12:	Answer to Problem 13:	Answer to Problem 14:
$	cubic centimeters	$

The Euclidean Algorithm, when applied to the numbers 11 and 60, returns the result that

$$11 \times 11 - 2 \times 60 = 1$$

which (algorithm or not) is surely true, because

$$121 - 120 = 1$$

The fact noted above is useful for the *most sophisticated* way of solving some of the problems below, but you are free to solve the problems in any way you can.

A conventional clock has an hour-hand, which undergoes one revolution every 12 hours, and a minute-hand, which undergoes one revolution every 1 hour. The clock is provided with 60 evenly spaced tick-marks. At 12:00, both hands are exactly straight up.

15. At a certain time, both the hour-hand and the minute-hand are exactly at tick-marks, and the minute-hand is one tick-mark more advanced than the hour-hand. What time does the clock read?

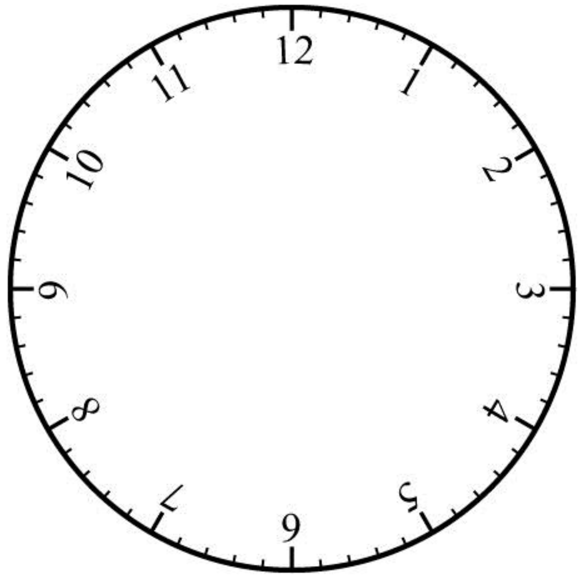
16. At another time, both the hour-hand and the minute-hand are exactly at tick-marks, but the minute hand is one tick-mark behind the hour-hand. What time does the clock read?

17. At yet a third time, the minute-hand is exactly 48° ahead of the hour-hand, and both the hour-hand and the minute-hand are exactly at tick-marks. What time does the clock read?

18. Surely at 9:00, the minute-hand is advanced 90° ahead of the hour-hand. When will the minute-hand next be exactly 90° ahead of the hour-hand, rounded expertly to the nearest second? Neither hour-hand nor minute-hand will be exactly at tick-marks.

— In order to receive credit, answers must appear in these boxes and be in the form specified. —

Answer to Problem 15:	Answer to Problem 16:	Answer to Problem 17:	Answer to Problem 18:
_____ : _____	_____ : _____	_____ : _____	_____ : _____ : _____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