THE · 34TH · PEE – DEE · REGIONAL · HIGH – SCHOOL

MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED · BY · FRANCIS · MARION · UNIVERSITY

AND · THE · PEE · DEE · EDUCATION · CENTER

TUESDAY · 2010 · DECEMBER · 07

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name *as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones are also strictly forbidden.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 200 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	Acceptable
One must perform all arithmetic that evaluates to an integer.	$2^2\,3^3\,5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$
All ratios must be written as a pure number in conventional notation. Translate ":" as "/" and, if necessary, simplify the resulting fraction.	1 : 2	$\frac{1}{2}$

↑ Name. (Print neatly and fully.)

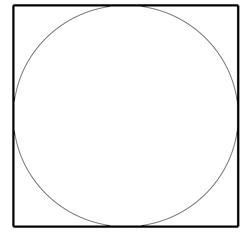
↑ High School. ↑ County.

— For official use only —

Page **1**. (#1, 2, 3, 4)	Page **2**. (#5, 6, 7, 8)
Page **3**. (# 9,10,11,12)	Page **4**. (#13, 14, 15)
Page **5**. (#16, 17, 18)	Page **6**. (#19, 20)
Total Correct	**Weighted Sum***

Awards

* Used only in tie-breaking.



1. A square is circumscribed about a circle of unit radius.
What is the perimeter of the square?

2. Perform the product, working entirely base ten: 10101×1101

3. Perform the product, working entirely base two: 10101×1101

4. Solve for x: $1+\dfrac{2}{3+\dfrac{4}{x}} = 1+\dfrac{2}{3+\dfrac{5}{6}}$.

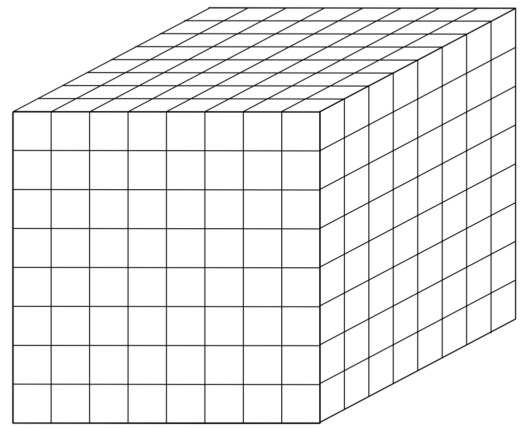
— Answers to problems <u>must</u> appear in these boxes in order to receive credit. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**	**Answer to Problem 4:**
units	$10101_{\text{base ten}} \times 1101_{\text{base ten}}$ $=$ base ten	$10101_{\text{base two}} \times 1101_{\text{base two}}$ $=$ base two	$x =$

5. Pipe A can drain a pool in 2 hours. Pipe B can drain a pool in 4 hours. In how many hours can pipes A and B drain the pool if they work together? (Answer as a fraction properly simplified.)

6. As in Problem 5, Pipe A can drain a pool in 2 hours and Pipe B can drain a pool in 4 hours. When Pipe C is also used, Pipes A, B, and C, working together, can drain the pool in 45 minutes. How long would it take Pipe C to drain the pool, working alone? (Answer as a fraction properly simplified.)

7. At right is shown an $8 \times 8 \times 8$ block of cubes. Suppose that all the cubes on the exterior are painted red. How many cubes have red paint on them?

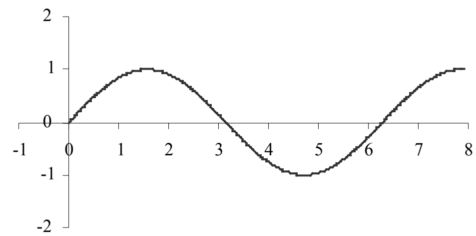


8. The cubes painted red in Problem 7 are removed and placed upon a red table. The exterior cubes of the remaining block are then painted blue and these cubes with blue paint on them are removed and placed upon a blue table. The process repeats, alternating red and blue, until there are no cubes left to paint and move to a table. How many cubes are on the red table at the end of the process?

— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 5:	Answer to Problem 6:	Answer to Problem 7:	Answer to Problem 8:
hours	hours		

9. The first crossing of the graph of $\{y = \sin x\}$ along the positive x-axis is between the positive integers 3 and 4, and the second crossing of the graph of $\{y = \sin x\}$ along the positive x-axis is between the positive integers 6 and 7. Between which two consecutive positive integers is the *hundredth* crossing of the graph of $\{y = \sin x\}$ along the positive x-axis?



10. Mary is looking at the numbers between 1 and 1000, inclusive. How many are divisible by 3?

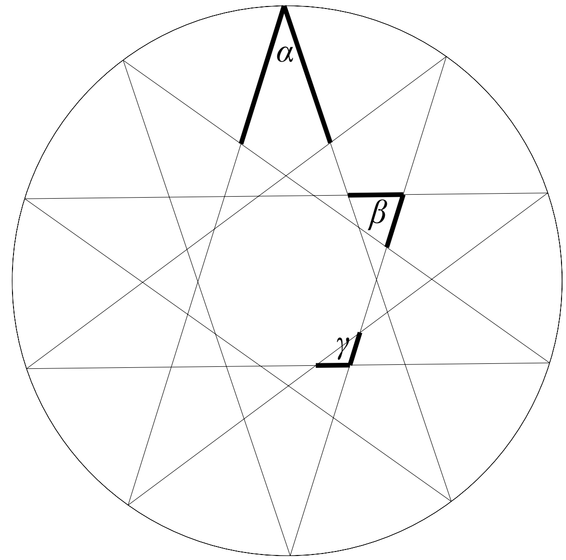
11. Mary is still looking at the numbers between 1 and 1000, inclusive. How many are divisible by 3 or 4?

12. Mary is *still* looking at the numbers between 1 and 1000, inclusive. How many are divisible by 3, 4, or 5?

— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 9:	Answer to Problem 10:	Answer to Problem 11:	Answer to Problem 12:
Between and			

13. In the diagram at right, a ten-pointed star is formed by joining each of ten points equally spaced along a circle to the point four segments along the circle away from it. What is the measure of the angle marked α? You may answer in either degrees or radians, here and in the next two problems.



14. In the same diagram, what is the measure of the angle marked β?

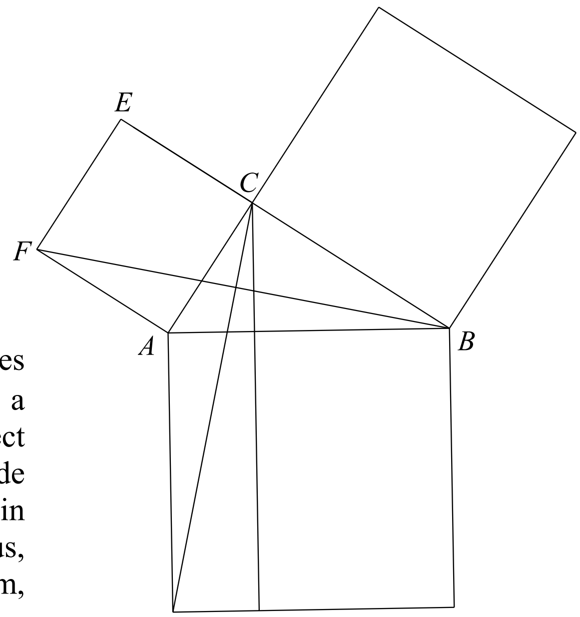
15. In the same diagram, what is the measure of the angle marked γ?

— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 13:	Answer to Problem 14:	Answer to Problem 15:
$\alpha =$	$\beta =$	$\gamma =$

16. Simplify completely: $\dfrac{1}{\sqrt{3}-1} - \dfrac{\sqrt{3}-7}{2}$

17. The diagram to the right shows the famous "windmill" diagram of Euclid's proof of the Pythagorean Theorem, applied to a 3-4-5 right triangle. In the diagram, $AC = 3$, $BC = 4$, and $AB = 5$, and $ACEF$ forms a square; furthermore, $\angle ACB$ is a right angle. What is the area of triangle AFB?



18. Plato's Perfect Ice Cream Shop sells ice-cream cones that consist of three scoops of ice cream sitting in a perfect cone. The scoops of ice cream are perfect spheres as well, and sit tangent to one another inside the cone. If the topmost scoop is a sphere 3 cm in radius, and the scoop underneath it is 2 cm in radius, what is the radius of the smallest scoop of ice cream, that is, the scoop at the bottom of the cone?

— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 16:	**Answer to Problem 17:**	**Answer to Problem 18:**
	square units	centimeters

Recall that a linear function of x is by definition a function of the form $ax + b$, where a and b are fixed real numbers. Recall also that, for any functions f and g, $f \circ g$ is the function defined by the fact that $(f \circ g)(x) = f(g(x))$.

In the following problems, let h and k be linear functions with

$$h(x) = 3x - 4 \qquad \text{and} \qquad k(x) = 4x + 5.$$

19. Find a linear function f so that $f \circ h = k$. That is, find a linear function $f(x)$ so that $(f \circ h)(x) = k(x)$ for all real numbers x.

20. Find a linear function g so that $g \circ g = k$. That is, find a linear function $g(x)$ so that $(g \circ g)(x) = k(x)$ for all real numbers x. Two answers are possible; answer any *one* of them.

Note well: *Remember throughout to perform all simplifications indicated on the front of this competition and to have answers in the proper boxes. The judges are instructed to reject answers that do not adhere to these protocols even if they are otherwise right.*

Answer to Problem 19:	**Answer to Problem 20:**
$f(x) =$	$g(x) =$

Answer to Ultimate Tiebreaker: