

THE 30TH PEE – DEE REGIONAL HIGH – SCHOOL
MATHEMATICS TOURNAMENT

Written Competition

SPONSORED BY FRANCIS MARION UNIVERSITY
AND THE PEE - DEE EDUCATION CENTER
TUESDAY 2006 DECEMBER 05

Instructions

- Do not turn over this page until instructed to do so.
- Neatly print (not sign) your name *as you wish it to appear if you are given an award*.
- During the examination, no calculators are allowed.
- Answers with fractions should be reported in lowest form. Answers involving π should be written as such (that is, do *not* use 3.14 as an approximation to π), and similarly for answers with square-roots.
- Place your final answers in the boxes on each page designated for that purpose. If you change your final answer, cross out the box and label where your final answer is clearly and emphatically.

Name (print neatly and fully):

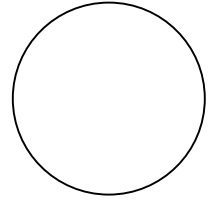
High School: _____

— *Do not write in these spaces.* —

Page 1. # 1, 2, 3	Page 2. # 4, 5, 6	Page 3. # 7, 8, 9		Total Number Correct
Page 4. # 10,11	Page 5. #12,13	Page 6. #14,15, 16		Weighted Total (used only for tie-breaking)

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1. A circle has an area of 1 square unit. What is its circumference?

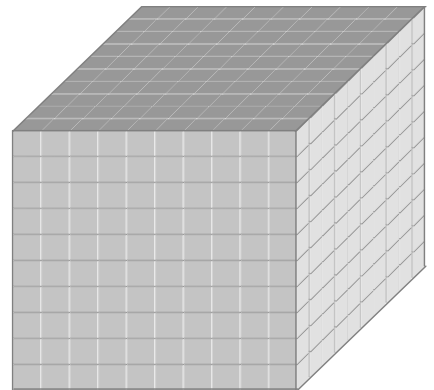


Answer to Problem 1:

2. Reexpress the fraction with its denominator rationalized: $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}}$

Answer to Problem 2:

3. A large cube is divided into 1000 sub-cubes, as shown. The six faces of the large cube are painted red. When it dries it is disassembled and those sub-cubes with one or three sides painted red are discarded. How many sub-cubes remain?



Answer to Problem 3:

4. Consider the following division: $\frac{0.\overline{5}}{2} = \frac{0.5555\dots}{2}$. Write the result as a decimal.

Answer to Problem 4:

5. Mary wrote all the numbers from 1 to 1000. How many times did she write the symbol “0” in making her list?

Answer to Problem 5:

6. Find the product of the polynomials: $(x^{16} + 1) \cdot (x^8 + 1) \cdot (x^4 + 1) \cdot (x^2 + 1) \cdot (x + 1) \cdot (x - 1)$

Answer to Problem 6:

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7. Three pipes feed water into a pool. The first pipe can fill the pool in 5 hours, the second pipe can fill the pool in 8 hours, and the third pipe can fill the pool in 20 hours. How long will it take the pool to fill if all three pipes are in operation? To gain credit for this problem, you must also express your answer in hours and minutes.

Answer to Problem 7:
hours + minutes

8. When an inch of rain falls on a square kilometer of land, how many liters of water fall from the sky? (Note: an inch is 2.54 centimeters long, and a liter contains 1000 cubic centimeters of volume.)

Answer to Problem 8:

9. At a Rotary Club meeting, 20 politicians decided to shake hands so that each politician shook hands with each other politician. How many handshakes took place?

Answer to Problem 9:

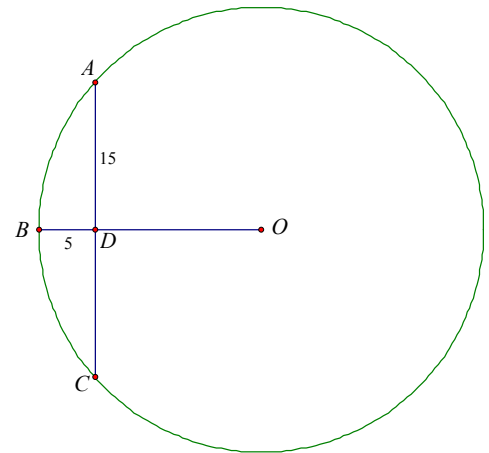
10. Find those real numbers A , B , and C so that the following equation is an identity in x :

$$\frac{7x+4}{x^3+3x^2+2x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

Answer to Problem 10:

$A =$	$B =$	$C =$
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11. In the diagram, O is the center of circle ABC , and \overline{OB} is a perpendicular bisector of the chord \overline{AC} , intersecting at D . Also, $\overline{AD} = 15$ and $\overline{BD} = 5$. What is the radius of the circle ABC ?



Answer to Problem 11:

Answer to Problem 11:

12. An 8×8 chessboard has one grain of wheat placed on its first square, two grains on the second square, four on its third square and so on. How many times more grain is on the fifth file than on the first file? (The first file of the chessboard is the set of squares marked {1, 9, 17, 25, 33, 41, 49, 57} and the fifth file of the chessboard is the set of squares marked {5, 13, 21, 29, 37, 45, 53, 61}.)

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38		40
41	42	43	44	45	46	47	48
49	50	51	52	53	54		56
57	58	59	60	61	62	63	64

Answer to Problem 12:

The fifth file has _____ times as much grain on it as the first file.

13. If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, what are the values of x and y ?

Answer to Problem 13:

$x =$ _____ and $y =$ _____

- 14.** Find the base b so that 554 base b is equal to 24^2 base b .
That is, find b so that 554_b is equal to 24_b^2

Answer to Problem 14:
$b =$

- 15.** Today is Tuesday. What day of the week will it be a 1000 days from now? (Note: *One* day from now it will be Wednesday.) Show work. Answers with no work will not receive credit on this problem even if the final answer is correct.

Answer to Problem 15:
Day of the week, written out.

- 16.** Today is December 05, 2006. What day will it be 1000 days from now? Answer in the form specified in the answer box.

Do the work for this problem on the next page and be *sure* to copy your final answer to this answer box. Otherwise you will not get credit.

Answer to Problem 16:
Month written out Day Year

Note for those unfamiliar with the calendar: Thirty days hath September, April, June, and November. All the rest have thirty-one, except for February alone, to which we twenty-eight assign, till leap year makes it twenty-nine. Also, all years divisible by 4 and not divisible by 100 are leap years, with the exception that those divisible by 400 *are* leap years.