

Questions from the Written Competition for the Spring 2004 Pee-Dee Regional High-School Mathematics Tournament

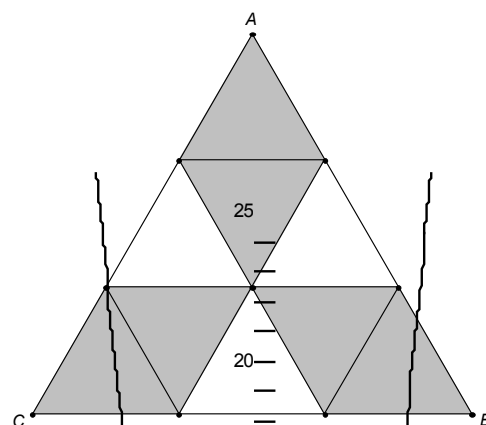
Sponsored by
The Pee Dee Education Center
and

The Department of Mathematics at Francis Marion University

Students had one hour to solve these problems. Space was permitted on the written competition for students to work their problems.



1. Triangle ABC is equilateral and all the smaller triangles are equilateral. If the area of triangle ABC is 51 square units, what is the area of the shaded part?

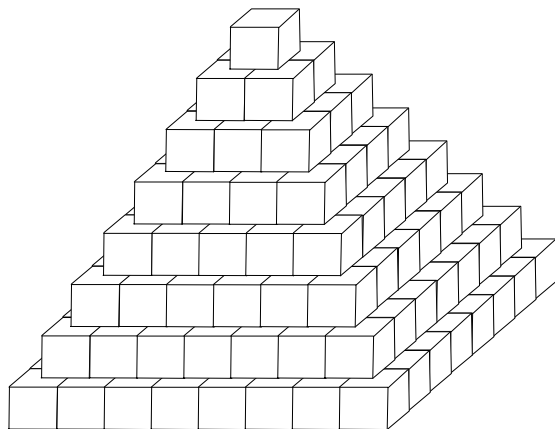
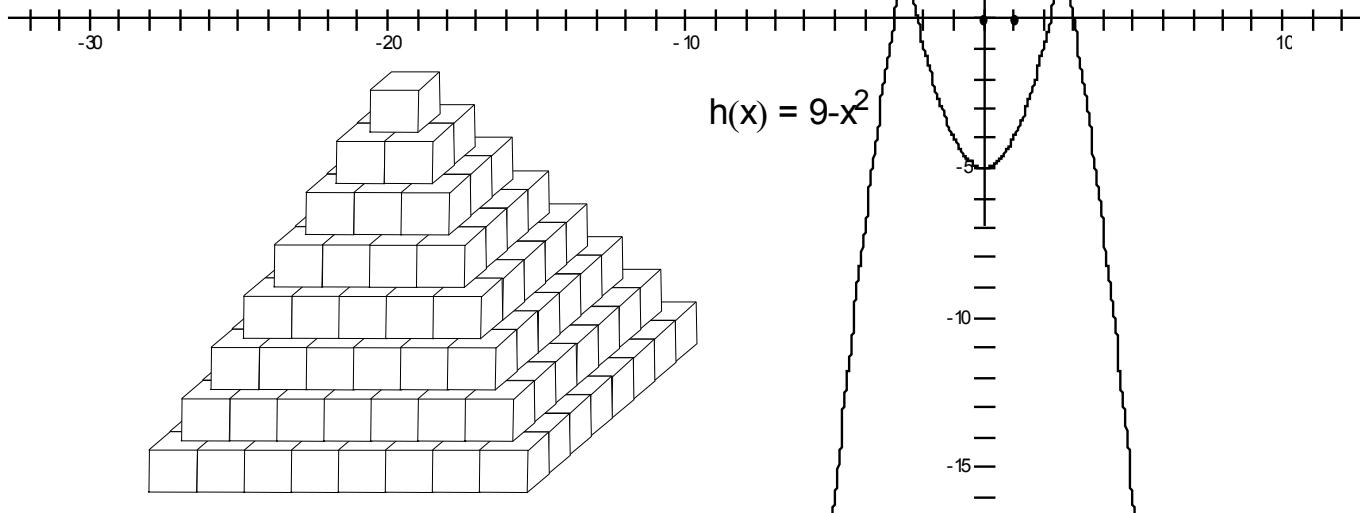


2. The two functions $f(x) = x^2 - 5$ and $h(x) = 9 - x^2$ intersect at points A and B. What is the distance between A and B?

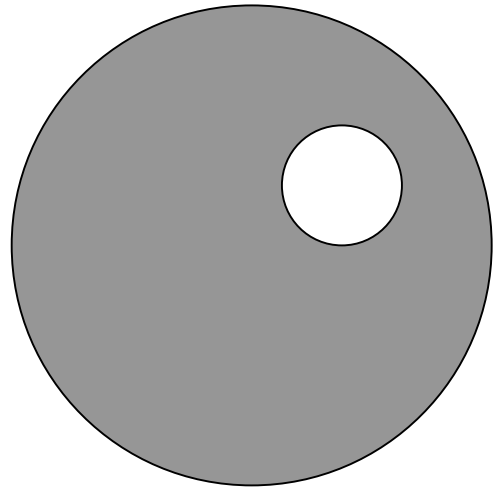
3. Assume that the pyramid displayed is completely filled with cubes all of the same size. How many cubes are contained in the pyramid?

4. Refer to the same pyramid as in question [3]. Suppose that the four triangular sides (not the base) are painted red. How many small cubes have red paint on them?

5. Sitting on a table are two one-liter bottles. Bottle 1 is full of water and bottle 2 is empty. One-half of the contents of bottle 1 are poured into bottle 2. Then one-half the contents of bottle two are poured into bottle 1. This procedure is repeated two more times. How much water is in each of the bottles?



6. In the diagram to the right the larger circle has four times the diameter of the smaller circle. The area of the large circle is 64 square units. What is the area of the shaded region?



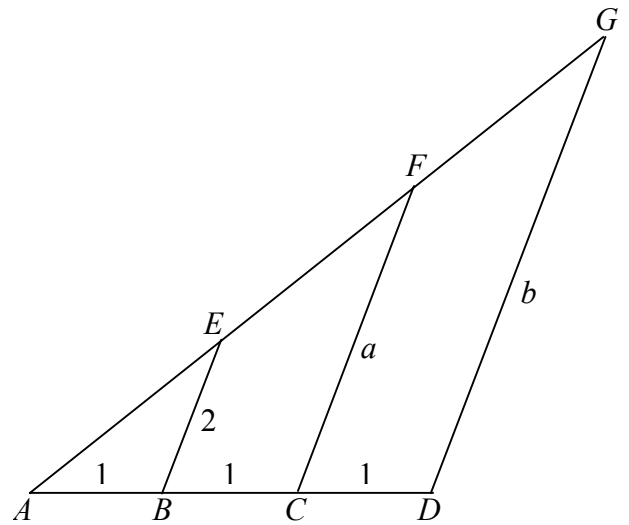
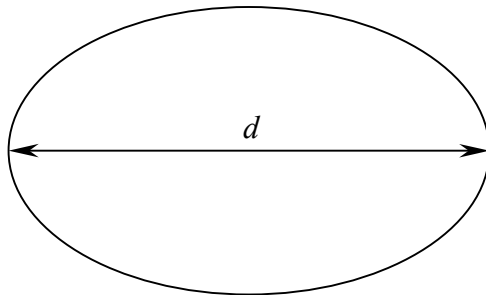
7. In the diagram to the right segments, \overline{BE} , \overline{CF} , and \overline{DG} are all parallel. The lengths of the segments are given in the diagram. Find the value of the of

$$2 + a + b.$$

8. The area of a circle is 36π . What is the circumference of the circle?
9. The length of the *major axis* of an ellipse is the distance d shown in the diagram. If an ellipse has an equation

$$9x^2 - 36x + 25y^2 + 50y - 164 = 0$$

what is the length if its major axis?



10. Simplify the expression
- $$\frac{(x^2 - 1)(x^4 - 1)(x^6 - 1)}{(x + 1)(x^2 + 1)(x^3 + 1)}.$$

11. If $\log(a) + \log(a^2) + \log(a^3) + \Lambda + \log(a^{10}) = 110$, what is the value of a ? (Note: the logarithms here are base ten.)

12. Write the following sum of repeating decimals as a repeating decimal.

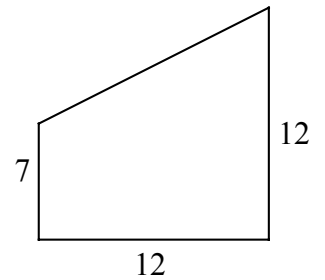
$$0.\overline{3} + 0.\overline{5} + 0.\overline{7}.$$

13. A person's salary is \$10,000. In year 1 he gets a 10% salary reduction. In year 2 he gets a 10% salary increase. What is his final salary?

14. Let $f(x) = \frac{1}{x^2}$. Simplify $f \circ f \circ f \circ f(x) = f(f(f(f(x))))$

- 15.** Lined up on a school hallway are 1000 lockers (numbered 1 through 1000). A student walks down the hall and opens every fifth locker. Then a second student goes to every seventh locker and opens the locker (if he finds it closed) or closes it (if he finds it open). How many lockers are now open?

- 16.** The trapezoid on the right has the lengths of three sides given. What is the perimeter of this trapezoid?



- 17.** A bathtub has four faucets. The time it takes each faucet by itself to fill a tub is given in the table to the right. If all four faucets are operating how long will it take the tub to fill (in minutes)? Express your answer as a rational number in lowest terms and as a decimal.

Faucet Number	Time to fill tub(minutes)
1	10
2	20
3	30
4	40

- 18.** In the diagram $\triangle ABC$ is a right triangle and each of the three quadrilaterals are squares. If the area of square $ACDE$ is 15 and the area of square $CBHI$ is 25, what is the area of square $AFGB$?

_____ ♦ _____

