THE · 42ND · PEE – DEE · REGIONAL · HIGH – SCHOOL
MATHEMATICS · TOURNAMENT

Written Competition

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MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER
TUESDAY · 2018 · DECEMBER · 04

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name in the space below
as you wish it to appear if you are given an award.

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored. If a problem specifies a certain form for an answer, then your answer *must* conform in order to receive credit.

Because the judges must score over 350 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	**Acceptable**
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$

Name of student
Name of high school
Awards

Page	Problems	Number Correct
2	1 2 3 4	
3	5 6 7 8	
4	9 10 11 12 13	
6	14 15 16 17	
7	18 19 20 21	
Total		

The time *noon* is written 1 2 : 0 0 : 0 0 *p.m.*, that is, complete with hours, minutes, and seconds. Write out these other times in this notation.

1. What is the time 100 seconds after noon?

2. What is the time 100 minutes after noon?

3. What is the time 100 hours after noon, and is the time "*a.m.*" or "*p.m.*"?

4. What is the time 155 hours, 73 minutes, and 45 seconds after 3:25:30 *p.m.*?

— In order to receive credit, answers must appear in these boxes and in the form specified. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**	**Answer to Problem4:**
__ __ : __ __ : __ __ *p.m.*	__ __ : __ __ : __ __ *p.m.*	__ __ : __ __ : __ __ __. __.	__ __ : __ __ : __ __ __. __.

A thousand is the number 10^3, and a million is a thousand thousand. A billion is a thousand million, and a trillion is a thousand billion.

5. In Fiscal Year 2017, the United States Government spent 4 trillion dollars. (The number has been rounded, of course, to make for a competition problem, but it is otherwise accurate.) There are about 320 million people in the United States, half of whom are wage-earners. What was the average expenditure of the United States Government that year *per wage-earner*, rounded expertly, if needed, to the nearest integer?

6. In Fiscal Year 2017, the United States Government borrowed 700 billion dollars. What fraction of its expenditures were paid with borrowed money? (Use information above as needed to complete the problem.) Answer as a percent to three-digit accuracy, by filling in the blanks in the answer box with Arabic numerals.

7. This number, 4 trillion: what is its square-root (rounded, if needed, to the nearest integer)?

$$\sqrt{4 \text{ trillion}} = ?$$

8. The common logarithm of 2 is 0.30103 (to five significant digits of accuracy). That is, $\log_{10}(2) = 0.30103$. This number, 4 trillion: what is its common logarithm?

$$\log_{10}(4 \text{ trillion}) = ?$$

Your answer must be in decimals to at least five significant digits of accuracy.

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 5:	Answer to Problem 6:	Answer to Problem 7:	Answer to Problem 8:
dollars per wage-earner	__ __ . __ %		

The following table of exact values for trigonometric functions of 15° may be of use for solving some or all of the problems on this page.

$\sin 15^\circ = \frac{1}{2} \cdot \sqrt{2-\sqrt{3}}$ $\quad$ $\csc 15^\circ = 2 \cdot \sqrt{2+\sqrt{3}}$

$\cos 15^\circ = \frac{1}{2} \cdot \sqrt{2+\sqrt{3}}$ $\quad$ $\sec 15^\circ = 2 \cdot \sqrt{2-\sqrt{3}}$

$\tan 15^\circ = 2-\sqrt{3}$ $\quad$ $\cot 15^\circ = 2+\sqrt{3}$

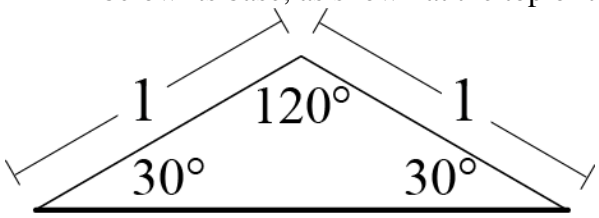
9. A certain isosceles triangle had shorter sides of length 1, and angles of 30° and 30° as its base angles. What was the length of its longest side?

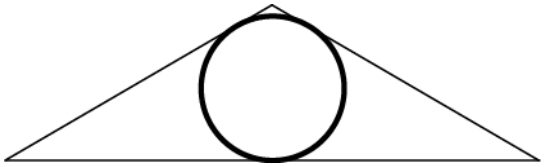
10. What is the radius of the circle inscribed in the triangle of Problem 9?

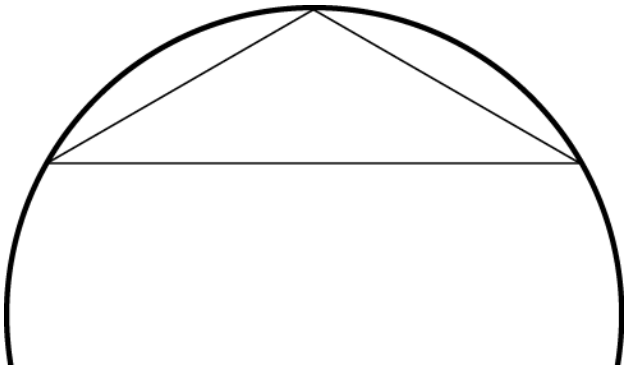
11. What is the radius of the circle circumscribed about the triangle of Problem 9?

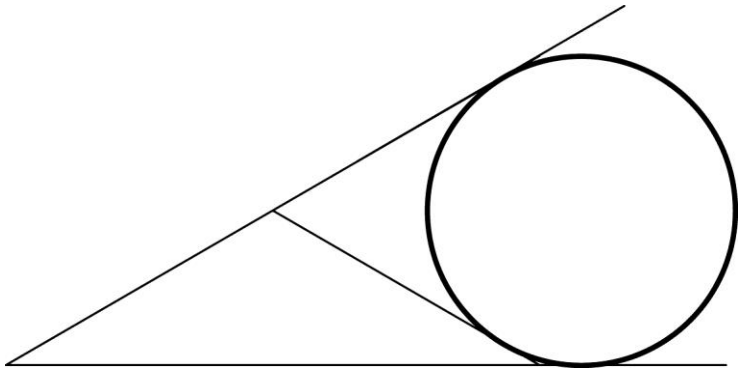
12. What is the radius of the circle externally tangent to the sides of the triangle in Problem 9 and to the right of it, as shown in the fourth drawing on this page?

13. What is the radius of the circle externally tangent to the triangle of Problem 9, but sitting to below its base, as shown at the top of the next page?



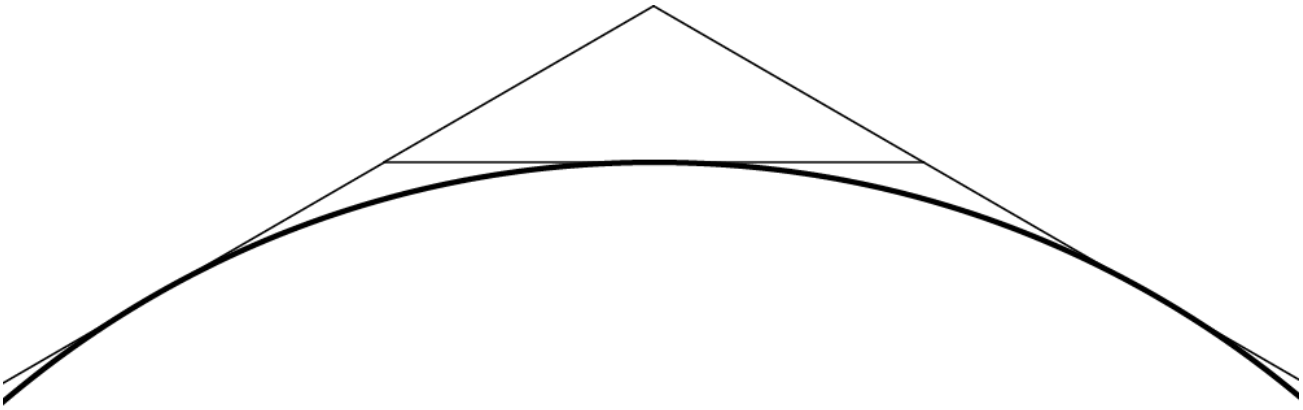






— In order to receive credit, answers must appear in these boxes and be completely simplified. —

Answer to Problem 9:	**Answer to Problem 10:**	**Answer to Problem 11:**	**Answer to Problem 12:**	**Answer to Problem13:**
units	units	units	units	units



— This competition continues for two more pages. —

For the function f on this page, the notation f^{-1} denotes the inverse function, not the reciprocal. Also, the notation f^n denotes the n^{th} *iterate* of f. For example, $f^3(x) = f(f(f(x)))$, not $(f(x))^3$.

14. "Just look at my function!" said Stud Man. "Add one and then square-root. For example, if I take the number 15 and do my function to it, I get the number 4." Stud Man applied his function twice to a number and ended up with the number 3. What number did he start with?

15. "What a wimpy function!" said Hot Rod derisively. "It takes big numbers and makes them little. I dislike Stud Man's function so much I'm going to find its inverse function, just so that I can undo his function whenever I need to." What function did Hot Rod create?

Put another way, find $f^{-1}(x)$, where f is Stud Man's function.

16. "I am not in agreement with Hot Rod's opinion," said Daisy Belle thoughtfully. "Though it is true that Stud Man's function makes large numbers smaller, it can make small numbers larger. For example, I have found a number that, after Stud Man's function is applied to it, grows by a factor of 2." What number is Daisy Belle referring to?

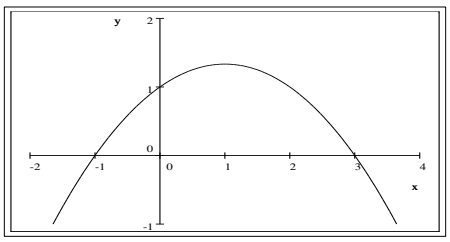
17. Daisy Belle then worked with Stud Man's function and made a discovery. "Look!" she said. "If I start with a positive number, large or small, and apply Stud Man's function to it repeatedly, the results always converge to the same real number, a number near and dear to my heart."

Put another way, for any positive real number x, what is $\lim_{n\to\infty} f^n(x)$?

— In order to receive credit, answers must appear in these boxes and in the form specified. —

Answer to Problem 14:	**Answer to Problem 15:**	**Answer to Problem 16:**	**Answer to Problem 17:**
	$f^{-1}(x)$ =		

For answers on this page, a polynomial must be written in full standard form, with blanks filled in with real numbers, which themselves must be properly simplified. For example, the polynomial $x^2 - 1$ must be reported as $\underline{1}\ x^2 + \underline{0}\ x + \underline{-1}$. *You may also use the next page for placing your work.*

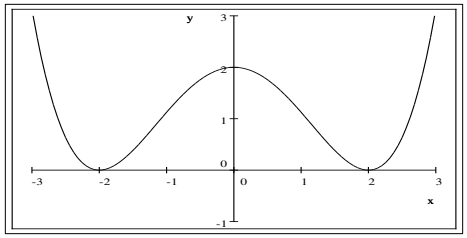


18. A quadratic polynomial has roots at –1 and at 3, and its graph crosses the *y*-axis at the point (0, 1). Its graph is shown above. What is the polynomial?

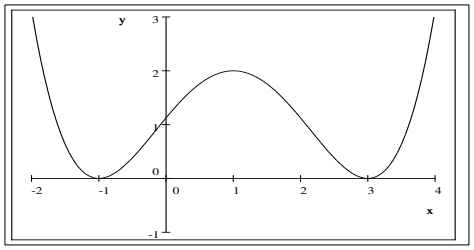
19. A cubic polynomial $g(x)$ has a table of inputs and outputs as shown at right. Its graph is *not* shown. What is the function $g(x)$?

x	$g(x)$
–1	3
0	5
1	4
2	8

20. A quartic polynomial has a repeated root at –2 and at +2. Its graph is perfectly symmetrical about the *y*-axis and it crosses the *y*-axis at the point (0, 2). What is the polynomial?



21. A quartic polynomial has a repeated root at –1 and at +3. Its graph is perfectly symmetrical about the line $\{x = 1\}$ and it crosses the line $\{x = 1\}$ at the point (1, 2). What is the polynomial?



— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 18:	**Answer to Problem 19:**
___ x^2 + ___ x + ___	$g(x)$ = ___ x^3 + ___ x^2 + ___ x + ___

Answer to Problem 20:	**Answer to Problem 21:**
___ x^4 + ___ x^3 + ___ x^2 + ___ x + ___	___ x^4 + ___ x^3 + ___ x^2 + ___ x + ___

If you are a high-school senior and would like to receive a Francis Marion University Application Fee Waiver Request Form, then print the word Yes in the box to the left. Such a form, if included in your application packet, will cause the fee to be waived if you apply for admission by 2019 March 31.

Request for a Francis Marion University Application Fee Waiver Form:	Answer to Ultimate Tiebreaker: